Maple 2018.2 Integration Test Results
on the problems in "0 Independent test suites"

Test results for the 48 problems in "Apostol Problems.txt"

Problem 7: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(8 x^3+27\right)^{2/3} dx$$

Optimal(type 2, 11 leaves, 1 step):

$$\frac{\left(8 x^3+27\right)^{5/3}}{40}$$

Result(type 2, 26 leaves):

$$\frac{(3+2 x) \left(4 x^2-6 x+9\right) \left(8 x^3+27\right)^{2/3}}{40}$$

Problem 9: Unable to integrate problem.

$$\int \frac{x}{\sqrt{1+x^2+\left(x^2+1\right)^{3/2}}} dx$$

Optimal(type 2, 26 leaves, 3 steps):

$$\frac{2 \sqrt{\left(x^2+1\right) \left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

Result(type 8, 18 leaves):

$$\int \frac{x}{\sqrt{1+x^2+\left(x^2+1\right)^{3/2}}} dx$$

Test results for the 12 problems in "Bondarenko Problems.txt"

Problem 3: Result is not expressed in closed-form.

$$\int \frac{\ln(1+x)}{x \sqrt{1+\sqrt{1+x}}} dx$$

Optimal(type 4, 224 leaves, ? steps):

$$-8 \operatorname{arctanh}\left(\sqrt{1+\sqrt{1+x}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{1+\sqrt{1+x}}\,\sqrt{2}}{2}\right) \ln(1+x) \sqrt{2}+2 \operatorname{arctanh}\left(\frac{\sqrt{2}}{2}\right) \ln\left(1-\sqrt{1+\sqrt{1+x}}\right) \sqrt{2}-2 \operatorname{arctanh}\left(\frac{\sqrt{2}}{2}\right) \ln\Big(1$$

$$+\sqrt{1+\sqrt{1+x}}\Big)\sqrt{2}+\mathrm{polylog}\left(2,-\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right)\sqrt{2}-\mathrm{polylog}\left(2,\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right)\sqrt{2}-\mathrm{polylog}\left(2,\right.$$

$$\left.-\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right)\sqrt{2}+\mathrm{polylog}\left(2,\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right)\sqrt{2}-\frac{2\ln(1+x)}{\sqrt{1+\sqrt{1+x}}}$$

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Result(type 7, 485 leaves):
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$$\ln(1+x)\ln\left(\sqrt{1+\sqrt{1+x}}-\sqrt{2}\right)\sqrt{2}-2\ln\left(\sqrt{1+\sqrt{1+x}}-\sqrt{2}\right)\ln\left(\frac{\sqrt{1+\sqrt{1+x}}-1}{\sqrt{2}-1}\right)\sqrt{2}-2\ln\left(\sqrt{1+\sqrt{1+x}}\right.$$

$$\left.-\sqrt{2}\right)\ln\left(\frac{1+\sqrt{1+\sqrt{1+x}}}{1+\sqrt{2}}\right)\sqrt{2}-2\,\mathrm{dilog}\left(\frac{\sqrt{1+\sqrt{1+x}}-1}{\sqrt{2}-1}\right)\sqrt{2}-2\,\mathrm{dilog}\left(\frac{1+\sqrt{1+\sqrt{1+x}}}{1+\sqrt{2}}\right)\sqrt{2}-\ln(1+x)\ln\left(\sqrt{1+\sqrt{1+x}}\right.$$

$$\left.+\sqrt{2}\right)\sqrt{2}+2\ln\left(\sqrt{1+\sqrt{1+x}}+\sqrt{2}\right)\ln\left(\frac{\sqrt{1+\sqrt{1+x}}-1}{-1-\sqrt{2}}\right)\sqrt{2}+2\ln\left(\sqrt{1+\sqrt{1+x}}+\sqrt{2}\right)\ln\left(\frac{1+\sqrt{1+\sqrt{1+x}}}{-\sqrt{2}+1}\right)\sqrt{2}$$

$$+2\,\mathrm{dilog}\left(\frac{\sqrt{1+\sqrt{1+x}}-1}{-1-\sqrt{2}}\right)\sqrt{2}+2\,\mathrm{dilog}\left(\frac{1+\sqrt{1+\sqrt{1+x}}}{-\sqrt{2}+1}\right)\sqrt{2}-4\left(\sum_{\_\alpha=RootOf(\_Z^2-2)}\frac{1}{8}\left(\_\alpha\left(\ln\left(\sqrt{1+\sqrt{1+x}}-\_\alpha\right)\ln(1+x)\right.\right.\right.$$

$$-2\,\mathrm{dilog}\left(\frac{\sqrt{1+\sqrt{1+x}}-1}{\_\alpha-1}\right)-2\ln\left(\sqrt{1+\sqrt{1+x}}-\_\alpha\right)\ln\left(\frac{\sqrt{1+\sqrt{1+x}}-1}{\_\alpha-1}\right)-2\,\mathrm{dilog}\left(\frac{1+\sqrt{1+\sqrt{1+x}}}{1+\_\alpha}\right)-2\ln\left(\sqrt{1+\sqrt{1+x}}\right.$$

$$\left.\left.\left.\left.-\_\alpha\right)\ln\left(\frac{1+\sqrt{1+\sqrt{1+x}}}{1+\_\alpha}\right)\right)\right)\right)-\frac{2\ln(1+x)}{\sqrt{1+\sqrt{1+x}}}-8\,\mathrm{arctanh}\left(\sqrt{1+\sqrt{1+x}}\right)$$

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Problem 5: Unable to integrate problem.
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$$\int\sqrt{1+\sqrt{x}+\sqrt{1+2x+2\sqrt{x}}}\,dx$$

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Optimal(type 2, 55 leaves, 2 steps):
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$$\frac{2\left(2+6x^{3/2}+\sqrt{x}-\left(2-\sqrt{x}\right)\sqrt{1+2x+2\sqrt{x}}\right)\sqrt{1+\sqrt{x}+\sqrt{1+2x+2\sqrt{x}}}}{15\sqrt{x}}$$

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Result(type 8, 21 leaves):
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$$\int\sqrt{1+\sqrt{x}+\sqrt{1+2x+2\sqrt{x}}}\,dx$$

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Problem 6: Unable to integrate problem.
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$$\int \sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2+2x+2\sqrt{2}\sqrt{x}}}\,dx$$

Optimal(type 2, 78 leaves, 3 steps):

$$\frac{2\sqrt{2}\left(4+3x^{3/2}\sqrt{2}+\sqrt{2}\sqrt{x}-\sqrt{2}\left(2\sqrt{2}-\sqrt{x}\right)\sqrt{1+x+\sqrt{2}\sqrt{x}}\right)\sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2}\sqrt{1+x+\sqrt{2}\sqrt{x}}}}{15\sqrt{x}}$$

Result(type 8, 26 leaves):

$$\int \sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2+2x+2\sqrt{2}\sqrt{x}}}\,dx$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x+\sqrt{1+x}}}{x^2}\,dx$$

Optimal(type 3, 59 leaves, 7 steps):

$$-\frac{\arctan\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)}{4}+\frac{3\,\operatorname{arctanh}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)}{4}-\frac{\sqrt{x+\sqrt{1+x}}}{x}$$

Result(type 3, 297 leaves):

$$-\frac{\left(\left(\sqrt{1+x}-1\right)^2+3\sqrt{1+x}-2\right)^{3/2}}{2\left(\sqrt{1+x}-1\right)}+\frac{3\sqrt{\left(\sqrt{1+x}-1\right)^2+3\sqrt{1+x}-2}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{\left(\sqrt{1+x}-1\right)^2+3\sqrt{1+x}-2}\right)}{2}$$
$$-\frac{3\,\operatorname{arctanh}\left(\frac{-1+3\sqrt{1+x}}{2\sqrt{\left(\sqrt{1+x}-1\right)^2+3\sqrt{1+x}-2}}\right)}{4}+\frac{\left(2\sqrt{1+x}+1\right)\sqrt{\left(\sqrt{1+x}-1\right)^2+3\sqrt{1+x}-2}}{4}-\frac{\left(\left(1+\sqrt{1+x}\right)^2-\sqrt{1+x}-2\right)^{3/2}}{2\left(1+\sqrt{1+x}\right)}$$
$$-\frac{\sqrt{\left(1+\sqrt{1+x}\right)^2-\sqrt{1+x}-2}}{4}-\frac{\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{\left(1+\sqrt{1+x}\right)^2-\sqrt{1+x}-2}\right)}{2}+\frac{\arctan\left(\frac{-3-\sqrt{1+x}}{2\sqrt{\left(1+\sqrt{1+x}\right)^2-\sqrt{1+x}-2}}\right)}{4}$$
$$+\frac{\left(2\sqrt{1+x}+1\right)\sqrt{\left(1+\sqrt{1+x}\right)^2-\sqrt{1+x}-2}}{4}$$

Problem 8: Unable to integrate problem.

$$\int \sqrt{\frac{1}{x}+\sqrt{1+\frac{1}{x}}}\, dx$$

Optimal(type 3, 72 leaves, 7 steps):

$$\frac{\arctan\left(\frac{3+\sqrt{1+\frac{1}{x}}}{2\sqrt{\frac{1}{x}+\sqrt{1+\frac{1}{x}}}}\right)}{4}-\frac{3\,\mathrm{arctanh}\left(\frac{1-3\sqrt{1+\frac{1}{x}}}{2\sqrt{\frac{1}{x}+\sqrt{1+\frac{1}{x}}}}\right)}{4}+x\sqrt{\frac{1}{x}+\sqrt{1+\frac{1}{x}}}$$

Result(type 8, 15 leaves):

$$\int \sqrt{\frac{1}{x}+\sqrt{1+\frac{1}{x}}}\, dx$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+e^{-x}}}{-e^{-x}+e^{x}}\, dx$$

Optimal(type 3, 19 leaves, 6 steps):

$$-\mathrm{arctanh}\left(\frac{\sqrt{1+e^{-x}}\,\sqrt{2}}{2}\right)\sqrt{2}$$

Result(type 3, 48 leaves):

$$-\frac{\sqrt{\frac{e^{x}+1}{e^{x}}}\, e^{x}\sqrt{2}\,\mathrm{arctanh}\left(\frac{(1+3\,e^{x})\sqrt{2}}{4\sqrt{(e^{x})^{2}+e^{x}}}\right)}{2\sqrt{e^{x}\,(e^{x}+1)}}$$

Problem 11: Unable to integrate problem.

$$\int \cosh(x)\,\ln\left(\cosh(x)^{2}+\sinh(x)\right)^{2} dx$$

Optimal(type 4, 312 leaves, 28 steps):

$$-2\ln\left(1+\sinh(x)+\sinh(x)^{2}\right)+8\sinh(x)-4\ln\left(1+\sinh(x)+\sinh(x)^{2}\right)\sinh(x)+\ln\left(1+\sinh(x)+\sinh(x)^{2}\right)^{2}\sinh(x)+\ln\left(1+\sinh(x)+\sinh(x)^{2}\right)\ln\Big(1$$
$$+2\sinh(x)-I\sqrt{3}\Big)\left(1-I\sqrt{3}\right)-\frac{\ln\left(1+2\sinh(x)-I\sqrt{3}\right)^{2}\left(1-I\sqrt{3}\right)}{2}-\ln\left(1+2\sinh(x)-I\sqrt{3}\right)\ln\left(-\frac{I}{6}\left(1+2\sinh(x)+I\sqrt{3}\right)\sqrt{3}\right)\Big(1$$
$$-I\sqrt{3}\Big)-\mathrm{polylog}\left(2,\frac{\left(I+2\,I\sinh(x)+\sqrt{3}\right)\sqrt{3}}{6}\right)\left(1-I\sqrt{3}\right)+\ln\left(1+\sinh(x)+\sinh(x)^{2}\right)\ln\left(1+2\sinh(x)+I\sqrt{3}\right)\left(1+I\sqrt{3}\right)$$

$$- \frac{\ln\left(1+2\sinh(x)+I\sqrt{3}\right)^2\left(1+I\sqrt{3}\right)}{2} - \ln\left(1+2\sinh(x)+I\sqrt{3}\right)\ln\left(\frac{I}{6}\left(1+2\sinh(x)-I\sqrt{3}\right)\sqrt{3}\right)\left(1+I\sqrt{3}\right) - \mathrm{polylog}\left(2, \frac{\left(-I-2I\sinh(x)+\sqrt{3}\right)\sqrt{3}}{6}\right)\left(1+I\sqrt{3}\right) - 4\arctan\left(\frac{(1+2\sinh(x))\sqrt{3}}{3}\right)\sqrt{3}$$

Result(type 8, 15 leaves):

$$\int \cosh(x)\ln\left(\cosh(x)^2+\sinh(x)\right)^2 dx$$

Test results for the 4 problems in "Bronstein Problems.txt"

Problem 4: Unable to integrate problem.

$$\int \frac{5x^2+3\left(e^x+x\right)^{1/3}+e^x\left(2x^2+3x\right)}{x\left(e^x+x\right)^{1/3}}\,dx$$

Optimal(type 3, 14 leaves, 8 steps):

$$3x\left(e^x+x\right)^{2/3}+3\ln(x)$$

Result(type 8, 38 leaves):

$$\int \frac{5x^2+3\left(e^x+x\right)^{1/3}+e^x\left(2x^2+3x\right)}{x\left(e^x+x\right)^{1/3}}\,dx$$

Test results for the 17 problems in "Charlwood Problems.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \arcsin(x)\ln(x)\,dx$$

Optimal(type 3, 45 leaves, 8 steps):

$$\mathrm{arctanh}\left(\sqrt{-x^2+1}\right) - x\arcsin(x)\left(1-\ln(x)\right) - 2\sqrt{-x^2+1} + \ln(x)\sqrt{-x^2+1}$$

Result(type 3, 91 leaves):

$$- \frac{2\left(\tan\left(\frac{\arcsin(x)}{2}\right)^2\ln\left(\frac{2\tan\left(\frac{\arcsin(x)}{2}\right)}{\tan\left(\frac{\arcsin(x)}{2}\right)^2+1}\right) - \arcsin(x)\tan\left(\frac{\arcsin(x)}{2}\right)\ln\left(\frac{2\tan\left(\frac{\arcsin(x)}{2}\right)}{\tan\left(\frac{\arcsin(x)}{2}\right)^2+1}\right) + \arcsin(x)\tan\left(\frac{\arcsin(x)}{2}\right)+2\right)}{\tan\left(\frac{\arcsin(x)}{2}\right)^2+1}$$

$$- \ln\left(\tan\left(\frac{\arcsin(x)}{2}\right)^2+1\right)$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int -\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)\, dx$$

Optimal(type 3, 49 leaves, ? steps):

$$-\left(\frac{3}{8}+x\right)\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}\,\sqrt{2}}{8}$$

Result(type 3, 250 leaves):

$$-\frac{1}{16\left(\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^2+1\right)^2\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^2}\left(\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^8\right.$$
$$-2\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^7+2\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^6-6\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^5+18\arcsin\left(\sqrt{x}\right.$$
$$\left.-\sqrt{1+x}\right)\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^4+6\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^3+2\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)^2$$
$$\left.+2\tan\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)+\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)\right)$$

Problem 4: Unable to integrate problem.

$$\int \frac{e^{\arcsin(x)}x^3}{\sqrt{-x^2+1}}\, dx$$

Optimal(type 3, 37 leaves, 5 steps):

$$\frac{e^{\arcsin(x)}\left(3x+x^3-3\sqrt{-x^2+1}-3x^2\sqrt{-x^2+1}\right)}{10}$$

Result(type 8, 18 leaves):

$$\int \frac{e^{\arcsin(x)}x^3}{\sqrt{-x^2+1}}\, dx$$

Problem 5: Unable to integrate problem.

$$\int \frac{\ln\left(x+\sqrt{x^2+1}\right)}{\left(-x^2+1\right)^{3/2}}\, dx$$

Optimal(type 3, 28 leaves, 3 steps):

$$-\frac{\arcsin\left(x^2\right)}{2}+\frac{x\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{-x^2+1}}$$

Result(type 8, 22 leaves):

$$\int \frac{\ln\left(x+\sqrt{x^2+1}\right)}{\left(-x^2+1\right)^{3/2}}\, dx$$

Problem 6: Unable to integrate problem.

$$\int \frac{\arcsin(x)}{\left(x^2+1\right)^{3/2}}\, dx$$

Optimal(type 3, 18 leaves, 3 steps):

$$-\frac{\arcsin\left(x^2\right)}{2}+\frac{x\arcsin(x)}{\sqrt{x^2+1}}$$

Result(type 8, 12 leaves):

$$\int \frac{\arcsin(x)}{\left(x^2+1\right)^{3/2}}\, dx$$

Problem 7: Unable to integrate problem.

$$\int \frac{\ln\left(x+\sqrt{x^2-1}\right)}{\left(x^2+1\right)^{3/2}}\, dx$$

Optimal(type 3, 26 leaves, 3 steps):

$$-\frac{\operatorname{arccosh}\left(x^2\right)}{2}+\frac{x\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}}$$

Result(type 8, 20 leaves):

$$\int \frac{\ln\left(x+\sqrt{x^2-1}\right)}{\left(x^2+1\right)^{3/2}}\, dx$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{\ln(x)}{x^2\sqrt{x^2-1}}\, dx$$

Optimal(type 3, 37 leaves, 4 steps):

$$-\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{\sqrt{x^2-1}}{x}+\frac{\ln(x)\sqrt{x^2-1}}{x}$$

Result(type 3, 88 leaves):

$$-\frac{\sqrt{-\mathrm{signum}(x^2-1)}\ \arcsin(x)}{\sqrt{\mathrm{signum}(x^2-1)}}+\frac{-\frac{\sqrt{-\mathrm{signum}(x^2-1)}\ \sqrt{-x^2+1}}{\sqrt{\mathrm{signum}(x^2-1)}}-\frac{\sqrt{-\mathrm{signum}(x^2-1)}\ \ln(x)\ \sqrt{-x^2+1}}{\sqrt{\mathrm{signum}(x^2-1)}}}{x}$$

Problem 10: Unable to integrate problem.

$$\int \frac{x\arctan(x)\ \ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,\mathrm{d}x$$

Optimal(type 3, 48 leaves, 4 steps):

$$-x\arctan(x)+\frac{\ln(x^2+1)}{2}-\frac{\ln\left(x+\sqrt{x^2+1}\right)^2}{2}+\arctan(x)\ \ln\left(x+\sqrt{x^2+1}\right)\ \sqrt{x^2+1}$$

Result(type 8, 23 leaves):

$$\int \frac{x\arctan(x)\ \ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,\mathrm{d}x$$

Problem 11: Unable to integrate problem.

$$\int \frac{\arctan(x)}{x^2\sqrt{-x^2+1}}\,\mathrm{d}x$$

Optimal(type 3, 48 leaves, 7 steps):

$$-\mathrm{arctanh}\left(\sqrt{-x^2+1}\right)+\mathrm{arctanh}\left(\frac{\sqrt{-x^2+1}\ \sqrt{2}}{2}\right)\sqrt{2}-\frac{\arctan(x)\ \sqrt{-x^2+1}}{x}$$

Result(type 8, 17 leaves):

$$\int \frac{\arctan(x)}{x^2\sqrt{-x^2+1}}\,\mathrm{d}x$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{x\,\mathrm{arcsec}(x)}{\sqrt{x^2-1}}\,\mathrm{d}x$$

Optimal(type 3, 21 leaves, 2 steps):

$$-\frac{x\ln(x)}{\sqrt{x^2}}+\mathrm{arcsec}(x)\ \sqrt{x^2-1}$$

Result(type 3, 96 leaves):

$$-\frac{2 I \sqrt{\frac{x^2-1}{x^2}}\, x \operatorname{arcsec}(x)}{\sqrt{x^2-1}}+\frac{\left(I \sqrt{\frac{x^2-1}{x^2}}\, x+x^2-1\right) \operatorname{arcsec}(x)}{\sqrt{x^2-1}}+\frac{\sqrt{\frac{x^2-1}{x^2}}\, x \ln\left(\left(\frac{1}{x}+I \sqrt{1-\frac{1}{x^2}}\right)^2+1\right)}{\sqrt{x^2-1}}$$

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Problem 13: Result unnecessarily involves higher level functions and more than twice size of optimal
antiderivative.
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$$\int \frac{-x^2+1}{(x^2+1)\sqrt{x^4+1}}\, dx$$

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Optimal(type 3, 18 leaves, 2 steps):
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$$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{2}$$

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Result(type 4, 111 leaves):
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$$-\frac{\sqrt{1-I x^2}\,\sqrt{1+I x^2}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{I\sqrt{2}}{2}\right), I\right)}{\left(\frac{\sqrt{2}}{2}+\frac{I\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{2\,(-1)^{3/4}\sqrt{1-I x^2}\,\sqrt{1+I x^2}\,\operatorname{EllipticPi}\left((-1)^{1/4} x, I, \frac{\sqrt{-I}}{(-1)^{1/4}}\right)}{\sqrt{x^4+1}}$$

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Problem 14: Unable to integrate problem.
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$$\int \ln(\sin(x))\sqrt{1+\sin(x)}\, dx$$

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Optimal(type 3, 36 leaves, 6 steps):
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$$-4 \operatorname{arctanh}\left(\frac{\cos(x)}{\sqrt{1+\sin(x)}}\right)+\frac{4\cos(x)}{\sqrt{1+\sin(x)}}-\frac{2\cos(x)\ln(\sin(x))}{\sqrt{1+\sin(x)}}$$

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Result(type 8, 12 leaves):
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$$\int \ln(\sin(x))\sqrt{1+\sin(x)}\, dx$$

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Problem 15: Unable to integrate problem.
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$$\int \sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}\, dx$$

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Optimal(type 3, 247 leaves, ? steps):
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$$\cot(x)\sqrt{2}\,\sqrt{-1+\sec(x)}\,\sqrt{1+\sec(x)}\left(\arctan\left(\frac{\left(-\sqrt{2}-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}\right)\sqrt{-2+2\sqrt{2}}}{2\sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}}\right)\sqrt{\sqrt{2}-1}\right.$$

$$+\operatorname{arctanh}\left(\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}}{\sqrt{2}-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}\right)\sqrt{\sqrt{2}-1}-\arctan\left(\frac{\left(-\sqrt{2}-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}\right)\sqrt{2+2\sqrt{2}}}{2\sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}}\right)\sqrt{1+\sqrt{2}}$$

$$\left.-\operatorname{arctanh}\left(\frac{\sqrt{-2+2\sqrt{2}}\,\sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}}{\sqrt{2}-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}\right)\sqrt{1+\sqrt{2}}\right)$$

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Result(type 8, 19 leaves):
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$$\int\sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}\,dx$$

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Problem 16: Result more than twice size of optimal antiderivative.
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$$\int\arctan\left(x\sqrt{x^2+1}\right)dx$$

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Optimal(type 3, 92 leaves, 12 steps):
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$$-\frac{\arctan\left(-\sqrt{3}+2\sqrt{x^2+1}\right)}{2}+x\arctan\left(x\sqrt{x^2+1}\right)-\frac{\arctan\left(\sqrt{3}+2\sqrt{x^2+1}\right)}{2}-\frac{\ln\left(2+x^2-\sqrt{3}\sqrt{x^2+1}\right)\sqrt{3}}{4}+\frac{\ln\left(2+x^2+\sqrt{3}\sqrt{x^2+1}\right)\sqrt{3}}{4}$$

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Result(type 3, 507 leaves):
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$$x\arctan\left(x\sqrt{x^2+1}\right)+\frac{\sqrt{2}\sqrt{\frac{2(x-1)^2}{(-1-x)^2}+2}\,\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x-1)^2}{(-1-x)^2}+2}\,\sqrt{3}}{2}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{(-1-x)^2}+1}{\left(\frac{x-1}{-1-x}+1\right)^2}}\left(\frac{x-1}{-1-x}+1\right)}+\frac{\sqrt{2}\sqrt{\frac{2(1+x)^2}{(1-x)^2}+2}\,\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(1+x)^2}{(1-x)^2}+2}\,\sqrt{3}}{2}\right)}{3\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+1}{\left(\frac{1+x}{1-x}+1\right)^2}}\left(\frac{1+x}{1-x}+1\right)}$$

$$-\frac{\sqrt{2}\sqrt{\frac{2(x-1)^2}{(-1-x)^2}+2}\left(\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x-1)^2}{(-1-x)^2}+2}\,\sqrt{3}}{2}\right)-3\arctan\left(\frac{\sqrt{\frac{2(x-1)^2}{(-1-x)^2}+2}\,(x-1)}{\left(\frac{(x-1)^2}{(-1-x)^2}+1\right)(-1-x)}\right)\right)}{12\sqrt{\frac{\frac{(x-1)^2}{(-1-x)^2}+1}{\left(\frac{x-1}{-1-x}+1\right)^2}}\left(\frac{x-1}{-1-x}+1\right)}$$

$$-\frac{\sqrt{2}\,\sqrt{\frac{2\,(1+x)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{2\,(1+x)^2}{(1-x)^2}+2}\,\sqrt{3}}{2}\right)-3\arctan\left(\frac{\sqrt{\frac{2\,(1+x)^2}{(1-x)^2}+2}\,(1+x)}{\left(\frac{(1+x)^2}{(1-x)^2}+1\right)(1-x)}\right)\right)}{12\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+1}{\left(\frac{1+x}{1-x}+1\right)^2}}\,\left(\frac{1+x}{1-x}+1\right)}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \arcsin\left(\frac{x}{\sqrt{-x^2+1}}\right)\,dx$$

Optimal(type 3, 25 leaves, 4 steps):

$$x\arcsin\left(\frac{x}{\sqrt{-x^2+1}}\right)+\arctan\left(\sqrt{-2\,x^2+1}\right)$$

Result(type 3, 137 leaves):

$$x\arcsin\left(\frac{x}{\sqrt{-x^2+1}}\right)+\frac{\sqrt{\frac{2\,x^2-1}{x^2-1}}\left(\sqrt{-2\,x^2+1}+\arctan\left(\frac{-1+2\,x}{\sqrt{-2\,x^2+1}}\right)-\arctan\left(\frac{1+2\,x}{\sqrt{-2\,x^2+1}}\right)\right)\sqrt{-x^2+1}}{\sqrt{-2\,x^2+1}\,\left(2+\sqrt{2}\right)\left(-2+\sqrt{2}\right)}+\frac{\sqrt{\frac{2\,x^2-1}{x^2-1}}\,\sqrt{-x^2+1}}{2}$$

Test results for the 77 problems in "Hearn Problems.txt"

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4-x^2+2}\,dx$$

Optimal(type 3, 132 leaves, 9 steps):

$$-\frac{\arctan\left(\frac{-2\,x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)\sqrt{14+28\sqrt{2}}}{28}+\frac{\arctan\left(\frac{2\,x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)\sqrt{14+28\sqrt{2}}}{28}-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{1+2\sqrt{2}}\right)}{4\sqrt{2+4\sqrt{2}}}$$

$$+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)}{4\sqrt{2+4\sqrt{2}}}$$

Result(type 3, 385 leaves):

$$
-\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}\,\sqrt{2}}{56}+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}}{14}+\frac{\arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)\left(1+2\sqrt{2}\right)\sqrt{2}}{28\sqrt{-1+2\sqrt{2}}}
$$

$$
-\frac{\arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)\left(1+2\sqrt{2}\right)}{7\sqrt{-1+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)\sqrt{2}}{2\sqrt{-1+2\sqrt{2}}}+\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}\,\sqrt{2}}{56}
$$

$$
-\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}}{14}+\frac{\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)\left(1+2\sqrt{2}\right)\sqrt{2}}{28\sqrt{-1+2\sqrt{2}}}-\frac{\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)\left(1+2\sqrt{2}\right)}{7\sqrt{-1+2\sqrt{2}}}
$$

$$
+\frac{\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)\sqrt{2}}{2\sqrt{-1+2\sqrt{2}}}
$$

Problem 17: Result is not expressed in closed-form.

$$
\int \frac{1}{x^8+1}\,dx
$$

Optimal(type 3, 239 leaves, 19 steps):

$$
-\frac{\ln\left(1+x^2-x\sqrt{2-\sqrt{2}}\right)\sqrt{2-\sqrt{2}}}{16}+\frac{\ln\left(1+x^2+x\sqrt{2-\sqrt{2}}\right)\sqrt{2-\sqrt{2}}}{16}-\frac{\arctan\left(\frac{-2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{4-2\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{4-2\sqrt{2}}}
$$

$$
-\frac{\ln\left(1+x^2-x\sqrt{2+\sqrt{2}}\right)\sqrt{2+\sqrt{2}}}{16}+\frac{\ln\left(1+x^2+x\sqrt{2+\sqrt{2}}\right)\sqrt{2+\sqrt{2}}}{16}-\frac{\arctan\left(\frac{-2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{4+2\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{4+2\sqrt{2}}}
$$

Result(type 7, 21 leaves):

$$
\frac{\left(\sum_{\_R=RootOf(\_Z^8+1)}\frac{\ln(x-\_R)}{\_R^7}\right)}{8}
$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int x^2 \sin(bx+a)^2 \, dx$$

Optimal(type 3, 63 leaves, 4 steps):

$$-\frac{x}{4b^2}+\frac{x^3}{6}+\frac{\cos(bx+a)\sin(bx+a)}{4b^3}-\frac{x^2\cos(bx+a)\sin(bx+a)}{2b}+\frac{x\sin(bx+a)^2}{2b^2}$$

Result(type 3, 157 leaves):

$$\frac{1}{b^3}\left((bx+a)^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)\cos(bx+a)^2}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}-\frac{(bx+a)^3}{3}-2a\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}+\frac{\sin(bx+a)^2}{4}\right)+a^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)\right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int x^2 \cos(bx+a)^2 \, dx$$

Optimal(type 3, 63 leaves, 4 steps):

$$-\frac{x}{4b^2}+\frac{x^3}{6}+\frac{x\cos(bx+a)^2}{2b^2}-\frac{\cos(bx+a)\sin(bx+a)}{4b^3}+\frac{x^2\cos(bx+a)\sin(bx+a)}{2b}$$

Result(type 3, 157 leaves):

$$\frac{1}{b^3}\left((bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{(bx+a)\cos(bx+a)^2}{2}-\frac{\cos(bx+a)\sin(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\frac{(bx+a)^3}{3}-2a\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{\sin(bx+a)^2}{4}\right)+a^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)\right)$$

Problem 66: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^{1/3}+\sqrt{x}} \, dx$$

Optimal(type 3, 24 leaves, 4 steps):

$$6x^{1/6}-3x^{1/3}-6\ln(1+x^{1/6})+2\sqrt{x}$$

Result(type 3, 91 leaves):

$$2\ln(x^{1/6}-1)-\ln(x^{1/3}+x^{1/6}+1)-2\ln(1+x^{1/6})+\ln(x^{1/3}-x^{1/6}+1)+2\sqrt{x}+\ln(\sqrt{x}-1)-\ln(\sqrt{x}+1)+6x^{1/6}-\ln(x-1)-2\ln(x^{1/3}-1)+\ln(x^{2/3}+x^{1/3}+1)-3x^{1/3}$$

Problem 74: Humongous result has more than 20000 leaves.

$$\int \frac{2x^6+4x^5+7x^4-3x^3-x^2-8x-8}{(2x^2-1)^2\sqrt{x^4+4x^3+2x^2+1}}\,dx$$

Optimal(type 3, 88 leaves, ? steps):

$$-\operatorname{arctanh}\left(\frac{x(2+x)(33x^3+27x^2-x+7)}{(31x^3+37x^2+2)\sqrt{x^4+4x^3+2x^2+1}}\right)+\frac{(1+2x)\sqrt{x^4+4x^3+2x^2+1}}{2(2x^2-1)}$$

Result(type ?, 1197350 leaves): Display of huge result suppressed!

Problem 76: Result more than twice size of optimal antiderivative.

$$\int \frac{\pi^2(4mc^9-3mc^8-48mc^7x+24mc^6x-144mc^5x^2+176mc^3x^3-24mc^2x^3+12mcx^4+3x^4)+12mc^3\pi^2(-12mc^2+3mc-8x)x^2\ln\left(\frac{x}{mc^2}\right)}{384\,e^{\frac{x}{y}}x^2}\,dx$$

Optimal(type 4, 304 leaves, 20 steps):

$$\frac{(3-4mc)mc^8\pi^2}{384\,e^{\frac{x}{y}}x}+\frac{3mc^5\pi^2 y}{8\,e^{\frac{x}{y}}}+\frac{(3-22mc)mc^2\pi^2 xy}{48\,e^{\frac{x}{y}}}-\frac{(1+4mc)\pi^2x^2y}{128\,e^{\frac{x}{y}}}+\frac{(3-22mc)mc^2\pi^2y^2}{48\,e^{\frac{x}{y}}}+\frac{mc^3\pi^2y^2}{4\,e^{\frac{x}{y}}}-\frac{(1+4mc)\pi^2xy^2}{64\,e^{\frac{x}{y}}}$$
$$-\frac{(1+4mc)\pi^2y^3}{64\,e^{\frac{x}{y}}}+\frac{(1-2mc)mc^6\pi^2\operatorname{Ei}\left(-\frac{x}{y}\right)}{16}+\frac{(3-4mc)mc^8\pi^2\operatorname{Ei}\left(-\frac{x}{y}\right)}{384y}+\frac{mc^3\pi^2(-12mc^2+3mc-8y)y\operatorname{Ei}\left(-\frac{x}{y}\right)}{32}$$
$$-\frac{mc^3\pi^2(3(1-4mc)mc-8x)y\ln\left(\frac{x}{mc^2}\right)}{32\,e^{\frac{x}{y}}}+\frac{mc^3\pi^2y^2\ln\left(\frac{x}{mc^2}\right)}{4\,e^{\frac{x}{y}}}$$

Result(type 4, 1355 leaves):

$$-\frac{\pi^2ymc\,e^{-\frac{x}{y}}x^2}{32}-\frac{\pi^2y^2mcx\,e^{-\frac{x}{y}}}{16}-\frac{3\pi^2y\,e^{-\frac{x}{y}}\ln(mc)mc^5}{4}+\frac{3\pi^2y\,e^{-\frac{x}{y}}\ln(mc)mc^4}{16}-\frac{\pi^2y^2\ln(mc)mc^3\,e^{-\frac{x}{y}}}{2}+\frac{\pi^2ymc^2x\,e^{-\frac{x}{y}}}{16}-\frac{11\pi^2ymc^3x\,e^{-\frac{x}{y}}}{24}$$
$$+\frac{\pi^2mc^8\,e^{-\frac{x}{y}}}{128x}-\frac{\pi^2mc^9\,e^{-\frac{x}{y}}}{96x}-\frac{\pi^2mc^8\operatorname{Ei}_1\left(\frac{x}{y}\right)}{128y}+\frac{\pi^2mc^9\operatorname{Ei}_1\left(\frac{x}{y}\right)}{96y}-\frac{\pi^2y^3mc\,e^{-\frac{x}{y}}}{16}-\frac{\pi^2y\,e^{-\frac{x}{y}}x^2}{128}-\frac{\pi^2y^2x\,e^{-\frac{x}{y}}}{64}+\frac{\pi^2y^2mc^2\,e^{-\frac{x}{y}}}{16}+\frac{3\pi^2y\,e^{-\frac{x}{y}}mc^5}{8}$$
$$-\frac{I\pi^3y^2mc^3\operatorname{csgn}(Imc)\operatorname{csgn}(Imc^2)^2\,e^{-\frac{x}{y}}}{4}+\frac{I\pi^3y^2mc^3\operatorname{csgn}(Imc)^2\operatorname{csgn}(Imc^2)\,e^{-\frac{x}{y}}}{8}-\frac{3I\pi^3y\,e^{-\frac{x}{y}}mc^4\operatorname{csgn}(Imc)^2\operatorname{csgn}(Imc^2)}{64}$$
$$+\frac{3I\pi^3y\,e^{-\frac{x}{y}}mc^4\operatorname{csgn}(Imc)\operatorname{csgn}(Imc^2)^2}{32}-\frac{3I\pi^3y\,e^{-\frac{x}{y}}mc^4\operatorname{csgn}(Ix)\operatorname{csgn}\left(\frac{Ix}{mc^2}\right)^2}{64}+\frac{3I\pi^3y\,e^{-\frac{x}{y}}mc^5\operatorname{csgn}\left(\frac{I}{mc^2}\right)\operatorname{csgn}\left(\frac{Ix}{mc^2}\right)^2}{16}$$

$$+\frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^5 \operatorname{csgn}(I mc)^2 \operatorname{csgn}(I mc^2)}{16} - \frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^5 \operatorname{csgn}(I mc) \operatorname{csgn}(I mc^2)^2}{8} + \frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^5 \operatorname{csgn}(I x) \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^2}{16}$$

$$-\frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^4 \operatorname{csgn}\left(\frac{I}{mc^2}\right) \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^2}{64} - \frac{I\pi^3 y mc^3 \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^3 x e^{-\frac{x}{y}}}{8} + \frac{I\pi^3 y^2 mc^3 \operatorname{csgn}\left(\frac{I}{mc^2}\right) \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^2 e^{-\frac{x}{y}}}{8}$$

$$+\frac{I\pi^3 y^2 mc^3 \operatorname{csgn}(I x) \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^2 e^{-\frac{x}{y}}}{8} + \frac{I\pi^3 y mc^3 \operatorname{csgn}(I mc^2)^3 x e^{-\frac{x}{y}}}{8} - \frac{\pi^2 y \ln(mc) mc^3 x e^{-\frac{x}{y}}}{2} - \frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^4 \operatorname{csgn}(I mc^2)^3}{64}$$

$$+\frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^4 \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^3}{64} + \frac{I\pi^3 y^2 mc^3 \operatorname{csgn}(I mc^2)^3 e^{-\frac{x}{y}}}{8} - \frac{I\pi^3 y^2 mc^3 \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^3 e^{-\frac{x}{y}}}{8} + \frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^5 \operatorname{csgn}(I mc^2)^3}{16}$$

$$-\frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^5 \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^3}{16} - \frac{5 mc^3 \pi^2 y^2 e^{-\frac{x}{y}}}{24} - \frac{\pi^2 mc^6 \operatorname{Ei}_1\left(\frac{x}{y}\right)}{16} + \frac{\pi^2 mc^7 \operatorname{Ei}_1\left(\frac{x}{y}\right)}{8} - \frac{\pi^2 y^3 e^{-\frac{x}{y}}}{64}$$

$$+\frac{\left(144\pi^2 mc^5 y - 36\pi^2 mc^4 y + 96\pi^2 mc^3 x y + 96\pi^2 mc^3 y^2\right) e^{-\frac{x}{y}} \ln(x)}{384} - \frac{I\pi^3 y mc^3 \operatorname{csgn}\left(\frac{I}{mc^2}\right) \operatorname{csgn}(I x) \operatorname{csgn}\left(\frac{I x}{mc^2}\right) x e^{-\frac{x}{y}}}{8} + \frac{\pi^2 mc^3 y^2 \operatorname{Ei}_1\left(\frac{x}{y}\right)}{4}$$

$$-\frac{3\pi^2 mc^4 y \operatorname{Ei}_1\left(\frac{x}{y}\right)}{32} + \frac{3\pi^2 mc^5 y \operatorname{Ei}_1\left(\frac{x}{y}\right)}{8} + \frac{I\pi^3 y mc^3 \operatorname{csgn}(I x) \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^2 x e^{-\frac{x}{y}}}{8} - \frac{I\pi^3 y^2 mc^3 \operatorname{csgn}\left(\frac{I}{mc^2}\right) \operatorname{csgn}(I x) \operatorname{csgn}\left(\frac{I x}{mc^2}\right) e^{-\frac{x}{y}}}{8}$$

$$-\frac{I\pi^3 y mc^3 \operatorname{csgn}(I mc) \operatorname{csgn}(I mc^2)^2 x e^{-\frac{x}{y}}}{4} + \frac{I\pi^3 y mc^3 \operatorname{csgn}(I mc)^2 \operatorname{csgn}(I mc^2) x e^{-\frac{x}{y}}}{8} - \frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^5 \operatorname{csgn}\left(\frac{I}{mc^2}\right) \operatorname{csgn}(I x) \operatorname{csgn}\left(\frac{I x}{mc^2}\right)}{16}$$

$$+\frac{3 I\pi^3 y e^{-\frac{x}{y}} mc^4 \operatorname{csgn}\left(\frac{I}{mc^2}\right) \operatorname{csgn}(I x) \operatorname{csgn}\left(\frac{I x}{mc^2}\right)}{64} + \frac{I\pi^3 y mc^3 \operatorname{csgn}\left(\frac{I}{mc^2}\right) \operatorname{csgn}\left(\frac{I x}{mc^2}\right)^2 x e^{-\frac{x}{y}}}{8}$$

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Test results for the 3 problems in "Hebisch Problems.txt"
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Problem 2: Unable to integrate problem.
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$$\int \frac{(2x^4 - x^3 + 3x^2 + 2x + 2) e^{\frac{x}{x^2+2}}}{x^3 + 2x} \, dx$$

Optimal(type 4, 27 leaves, ? steps):

$$e^{\frac{x}{x^2+2}} (x^2+2) + \mathrm{Ei}\left(\frac{x}{x^2+2}\right)$$

Result(type 8, 42 leaves):

$$\int \frac{(2x^4 - x^3 + 3x^2 + 2x + 2)\, e^{\frac{x}{x^2+2}}}{x^3 + 2x}\, dx$$

Test results for the 3 problems in "Jeffrey Problems.txt"

Test results for the 31 problems in "Moses Problems.txt"

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^{1/3} + \sqrt{x}}\, dx$$

Optimal(type 3, 24 leaves, 4 steps):

$$6x^{1/6} - 3x^{1/3} - 6\ln(1 + x^{1/6}) + 2\sqrt{x}$$

Result(type 3, 91 leaves):

$$2\ln(x^{1/6} - 1) - \ln(x^{1/3} + x^{1/6} + 1) - 2\ln(1 + x^{1/6}) + \ln(x^{1/3} - x^{1/6} + 1) + 2\sqrt{x} + \ln(\sqrt{x} - 1) - \ln(\sqrt{x} + 1) + 6x^{1/6} - \ln(x - 1) - 2\ln(x^{1/3} - 1) + \ln(x^{2/3} + x^{1/3} + 1) - 3x^{1/3}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{(-A^2 - B^2)\cos(z)^2}{B\left(1 - \frac{(A^2+B^2)\sin(z)^2}{B^2}\right)}\, dz$$

Optimal(type 3, 16 leaves, 5 steps):

$$-Bz - A\,\mathrm{arctanh}\left(\frac{A\tan(z)}{B}\right)$$

Result(type 3, 126 leaves):

$$-\frac{A^3\ln(A\tan(z) + B)}{2(A^2+B^2)} - \frac{AB^2\ln(A\tan(z) + B)}{2(A^2+B^2)} + \frac{A^3\ln(A\tan(z) - B)}{2(A^2+B^2)} + \frac{AB^2\ln(A\tan(z) - B)}{2(A^2+B^2)} - \frac{B\arctan(\tan(z))A^2}{A^2+B^2} - \frac{\arctan(\tan(z))B^3}{A^2+B^2}$$

Test results for the 101 problems in "Stewart Problems.txt"

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \sec(x)\,\tan(x)^5\,dx$$

Optimal(type 3, 15 leaves, 3 steps):

$$\sec(x) - \frac{2\sec(x)^3}{3} + \frac{\sec(x)^5}{5}$$

Result(type 3, 47 leaves):

$$\frac{\sin(x)^6}{5\cos(x)^5} - \frac{\sin(x)^6}{15\cos(x)^3} + \frac{\sin(x)^6}{5\cos(x)} + \frac{\left(\frac{8}{3} + \sin(x)^4 + \frac{4\sin(x)^2}{3}\right)\cos(x)}{5}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \sec(x)^3\tan(x)^3\,dx$$

Optimal(type 3, 13 leaves, 3 steps):

$$-\frac{\sec(x)^3}{3} + \frac{\sec(x)^5}{5}$$

Result(type 3, 41 leaves):

$$\frac{\sin(x)^4}{5\cos(x)^5} + \frac{\sin(x)^4}{15\cos(x)^3} - \frac{\sin(x)^4}{15\cos(x)} - \frac{\left(2 + \sin(x)^2\right)\cos(x)}{15}$$

Problem 98: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{\sqrt{x^{10}-2}}\,dx$$

Optimal(type 3, 14 leaves, 3 steps):

$$\frac{\operatorname{arctanh}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)}{5}$$

Result(type 3, 33 leaves):

$$\frac{\sqrt{-\operatorname{signum}\left(-1 + \frac{x^{10}}{2}\right)}\,\arcsin\left(\frac{x^5\sqrt{2}}{2}\right)}{5\sqrt{\operatorname{signum}\left(-1 + \frac{x^{10}}{2}\right)}}$$

Test results for the 193 problems in "Timofeev Problems.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{-b^2 x^2 + a^2}\, dx$$

Optimal(type 3, 14 leaves, 1 step):

$$\frac{\operatorname{arctanh}\left(\frac{b x}{a}\right)}{a b}$$

Result(type 3, 31 leaves):

$$-\frac{\ln(b x - a)}{2 a b} + \frac{\ln(b x + a)}{2 a b}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos(x)^3}{\sin(x)^4}\, dx$$

Optimal(type 3, 11 leaves, 2 steps):

$$-\frac{1}{3 \sin(x)^3} + \frac{1}{\sin(x)}$$

Result(type 3, 31 leaves):

$$-\frac{\cos(x)^4}{3 \sin(x)^3} + \frac{\cos(x)^4}{3 \sin(x)} + \frac{\left(2 + \cos(x)^2\right) \sin(x)}{3}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \ln(\cos(x))\, \sec(x)^2\, dx$$

Optimal(type 3, 12 leaves, 3 steps):

$$-x + \tan(x) + \ln(\cos(x))\tan(x)$$

Result(type 3, 60 leaves):

$$-\frac{2\, \mathrm{I}\, \mathrm{e}^{2\, \mathrm{I} x} \ln(2 \cos(x))}{1 + \mathrm{e}^{2\, \mathrm{I} x}} + \frac{2\, \mathrm{I}}{1 + \mathrm{e}^{2\, \mathrm{I} x}} + \mathrm{I} \ln\left(1 + \mathrm{e}^{2\, \mathrm{I} x}\right) - \frac{2\, \mathrm{I} \ln(2)}{1 + \mathrm{e}^{2\, \mathrm{I} x}}$$

Problem 38: Unable to integrate problem.

$$\int \frac{1}{x^m (a^3 + x^3)}\, dx$$

Optimal(type 5, 40 leaves, 1 step):

$$\frac{x^{1-m} \operatorname{hypergeom}\left(\left[1, \frac{1}{3} - \frac{m}{3}\right], \left[\frac{4}{3} - \frac{m}{3}\right], -\frac{x^3}{a^3}\right)}{a^3 (1 - m)}$$

Result(type 8, 17 leaves):

$$\int \frac{1}{x^m \left(a^3+x^3\right)}\, dx$$

Problem 40: Unable to integrate problem.

$$\int \frac{1}{x^m \left(a^4-x^4\right)}\, dx$$

Optimal(type 5, 39 leaves, 1 step):

$$\frac{x^{1-m}\mathrm{hypergeom}\left(\left[1, \frac{1}{4}-\frac{m}{4}\right], \left[\frac{5}{4}-\frac{m}{4}\right], \frac{x^4}{a^4}\right)}{a^4\,(1-m)}$$

Result(type 8, 19 leaves):

$$\int \frac{1}{x^m \left(a^4-x^4\right)}\, dx$$

Problem 41: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 \left(a^5+x^5\right)}\, dx$$

Optimal(type 3, 157 leaves, 7 steps):

$$-\frac{1}{a^5 x}+\frac{\ln(a+x)}{5\,a^6}-\frac{\ln\left(a^2+x^2-\frac{a\,x\left(-\sqrt{5}+1\right)}{2}\right)\left(-\sqrt{5}+1\right)}{20\,a^6}-\frac{\ln\left(a^2+x^2-\frac{a\,x\left(\sqrt{5}+1\right)}{2}\right)\left(\sqrt{5}+1\right)}{20\,a^6}$$
$$+\frac{\arctan\left(\frac{\left(-4\,x+a\left(\sqrt{5}+1\right)\right)\sqrt{50+10\sqrt{5}}}{20\,a}\right)\sqrt{10-2\sqrt{5}}}{10\,a^6}+\frac{\arctan\left(\frac{-4\,x+a\left(-\sqrt{5}+1\right)}{a\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}}{10\,a^6}$$

Result(type 7, 108 leaves):

$$\frac{\sum_{\_R=RootOf\left(\_Z^4-a\,\_Z^3+a^2\,\_Z^2-a^3\,\_Z+a^4\right)}\frac{\left(-\_R^3-3\,\_R^2\,a+2\,\_R\,a^2-a^3\right)\ln(x-\_R)}{4\,\_R^3-3\,\_R^2\,a+2\,\_R\,a^2-a^3}}{5\,a^6}+\frac{\ln(a+x)}{5\,a^6}-\frac{1}{a^5\,x}$$

Problem 51: Result more than twice size of optimal antiderivative.

$$\int \frac{c1\,x+b1}{\left(c\,x^2+2\,b\,x+a\right)^4}\, dx$$

Optimal(type 3, 162 leaves, 5 steps):

$$\frac{-b\,b1+a\,c1-(-b\,c1+b1\,c)\,x}{6\,(-a\,c+b^2)\,(c\,x^2+2\,b\,x+a)^3}+\frac{5\,(-b\,c1+b1\,c)\,(c\,x+b)}{24\,(-a\,c+b^2)^2\,(c\,x^2+2\,b\,x+a)^2}-\frac{5\,c\,(-b\,c1+b1\,c)\,(c\,x+b)}{16\,(-a\,c+b^2)^3\,(c\,x^2+2\,b\,x+a)}+\frac{5\,c^2\,(-b\,c1+b1\,c)\,\operatorname{arctanh}\left(\frac{c\,x+b}{\sqrt{-a\,c+b^2}}\right)}{16\,(-a\,c+b^2)^{7/2}}$$

Result(type 3, 404 leaves):

$$\frac{(-2\,b\,c1+2\,b1\,c)\,x+2\,b\,b1-2\,a\,c1}{3\,(4\,a\,c-4\,b^2)\,(c\,x^2+2\,b\,x+a)^3}-\frac{10\,c\,x\,b\,c1}{3\,(4\,a\,c-4\,b^2)^2\,(c\,x^2+2\,b\,x+a)^2}+\frac{10\,c^2\,x\,b1}{3\,(4\,a\,c-4\,b^2)^2\,(c\,x^2+2\,b\,x+a)^2}-\frac{10\,b^2\,c1}{3\,(4\,a\,c-4\,b^2)^2\,(c\,x^2+2\,b\,x+a)^2}$$
$$+\frac{10\,b\,b1\,c}{3\,(4\,a\,c-4\,b^2)^2\,(c\,x^2+2\,b\,x+a)^2}-\frac{20\,c^2\,x\,b\,c1}{(4\,a\,c-4\,b^2)^3\,(c\,x^2+2\,b\,x+a)}+\frac{20\,c^3\,x\,b1}{(4\,a\,c-4\,b^2)^3\,(c\,x^2+2\,b\,x+a)}-\frac{20\,c\,b^2\,c1}{(4\,a\,c-4\,b^2)^3\,(c\,x^2+2\,b\,x+a)}$$
$$+\frac{20\,c^2\,b\,b1}{(4\,a\,c-4\,b^2)^3\,(c\,x^2+2\,b\,x+a)}-\frac{20\,c^2\arctan\left(\frac{2\,c\,x+2\,b}{2\sqrt{a\,c-b^2}}\right)b\,c1}{(4\,a\,c-4\,b^2)^3\sqrt{a\,c-b^2}}+\frac{20\,c^3\arctan\left(\frac{2\,c\,x+2\,b}{2\sqrt{a\,c-b^2}}\right)b1}{(4\,a\,c-4\,b^2)^3\sqrt{a\,c-b^2}}$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(x-1)^{2/3}\,x^5}\,dx$$

Optimal(type 3, 75 leaves, 8 steps):

$$\frac{(x-1)^{1/3}}{4\,x^4}+\frac{11\,(x-1)^{1/3}}{36\,x^3}+\frac{11\,(x-1)^{1/3}}{27\,x^2}+\frac{55\,(x-1)^{1/3}}{81\,x}+\frac{55\ln\left(1+(x-1)^{1/3}\right)}{81}-\frac{55\ln(x)}{243}-\frac{110\arctan\left(\frac{\left(1-2\,(x-1)^{1/3}\right)\sqrt{3}}{3}\right)\sqrt{3}}{243}$$

Result(type 3, 157 leaves):

$$-\frac{1}{324\,\left(1+(x-1)^{1/3}\right)^4}-\frac{5}{243\,\left(1+(x-1)^{1/3}\right)^3}-\frac{20}{243\,\left(1+(x-1)^{1/3}\right)^2}-\frac{25}{81\,\left(1+(x-1)^{1/3}\right)}+\frac{110\ln\left(1+(x-1)^{1/3}\right)}{243}$$
$$-\frac{-75\,(x-1)^{7/3}+190\,(x-1)^2-350\,(x-1)^{5/3}+\frac{1157\,(x-1)^{4/3}}{4}-138\,x+\frac{149}{4}-116\,(x-1)^{2/3}+137\,(x-1)^{1/3}}{243\,\left((x-1)^{2/3}-(x-1)^{1/3}+1\right)^4}$$
$$-\frac{55\ln\left((x-1)^{2/3}-(x-1)^{1/3}+1\right)}{243}+\frac{110\sqrt{3}\arctan\left(\frac{\left(-1+2\,(x-1)^{1/3}\right)\sqrt{3}}{3}\right)}{243}$$

Problem 58: Unable to integrate problem.

$$\int\frac{x^2\,(-x^2+1)^{1/4}\sqrt{1+x}}{\sqrt{1-x}\,\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$$

Optimal(type 3, 219 leaves, 33 steps):

$$\frac{5(1-x)^{3/4}(1+x)^{1/4}}{16}-\frac{(1-x)^{1/4}(1+x)^{3/4}}{16}+\frac{(1-x)^{5/4}(1+x)^{3/4}}{24}+\frac{3\arctan\left(-1+\frac{(1-x)^{1/4}\sqrt{2}}{(1+x)^{1/4}}\right)\sqrt{2}}{16}$$
$$+\frac{3\arctan\left(1+\frac{(1-x)^{1/4}\sqrt{2}}{(1+x)^{1/4}}\right)\sqrt{2}}{16}+\frac{\ln\left(1-\frac{(1-x)^{1/4}\sqrt{2}}{(1+x)^{1/4}}+\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\sqrt{2}}{16}-\frac{\ln\left(1+\frac{(1-x)^{1/4}\sqrt{2}}{(1+x)^{1/4}}+\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\sqrt{2}}{16}$$
$$+\frac{7(-x^2+1)^{5/4}}{24\sqrt{1-x}}+\frac{x(-x^2+1)^{5/4}}{6\sqrt{1-x}}+\frac{(-x^2+1)^{5/4}\sqrt{1+x}}{6}$$

Result(type 8, 44 leaves):

$$\int\frac{x^2(-x^2+1)^{1/4}\sqrt{1+x}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$$

Problem 60: Unable to integrate problem.

$$\int\frac{((x-1)^2(1+x))^{1/3}}{x^2}\,dx$$

Optimal(type 3, 127 leaves, ? steps):

$$-\frac{((x-1)^2(1+x))^{1/3}}{x}+\frac{\ln(x)}{6}-\frac{2\ln(1+x)}{3}-\frac{3\ln\left(1+\frac{1-x}{((x-1)^2(1+x))^{1/3}}\right)}{2}-\frac{\ln\left(1+\frac{x-1}{((x-1)^2(1+x))^{1/3}}\right)}{2}$$
$$-\frac{\arctan\left(\frac{\left(1-\frac{2(x-1)}{((x-1)^2(1+x))^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\arctan\left(\frac{\left(1+\frac{2(x-1)}{((x-1)^2(1+x))^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}$$

Result(type 8, 73 leaves):

$$-\frac{((x-1)^2(1+x))^{1/3}}{x}+\frac{\left(\int\frac{3x+1}{3x((x-1)(1+x)^2)^{1/3}}\,dx\right)((x-1)^2(1+x))^{1/3}((x-1)(1+x)^2)^{1/3}}{(x-1)(1+x)}$$

Problem 66: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(x^4-1)\sqrt{x^2+2}}\,dx$$

Optimal(type 3, 31 leaves, 5 steps):

$$-\frac{\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{\sqrt{x^2+2}}\right)\sqrt{3}}{6}$$

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Result(type 3, 69 leaves):
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$$-\frac{\sqrt{3}\operatorname{arctanh}\left(\frac{(2x+4)\sqrt{3}}{6\sqrt{(x-1)^2+2x+1}}\right)}{12}+\frac{\sqrt{3}\operatorname{arctanh}\left(\frac{(4-2x)\sqrt{3}}{6\sqrt{(1+x)^2-2x+1}}\right)}{12}-\frac{\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)}{2}$$

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Problem 68: Result more than twice size of optimal antiderivative.
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$$\int\frac{1+2x}{(3x^2+4x+4)\sqrt{x^2+6x-1}}\,dx$$

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Optimal(type 3, 53 leaves, 5 steps):
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$$-\frac{\operatorname{arctanh}\left(\frac{(1+x)\sqrt{7}}{\sqrt{x^2+6x-1}}\right)\sqrt{7}}{21}-\frac{5\arctan\left(\frac{(2-x)\sqrt{7}\sqrt{2}}{4\sqrt{x^2+6x-1}}\right)\sqrt{14}}{84}$$

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Result(type 3, 157 leaves):
```

$$-\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\left(4\sqrt{7}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\sqrt{7}}{21}\right)-5\sqrt{14}\arctan\left(\frac{\sqrt{14}\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,(-2+x)}{4\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)(-1-x)}\right)\right)}{84\sqrt{-\frac{3\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)}{\left(\frac{-2+x}{-1-x}+1\right)^2}}\left(\frac{-2+x}{-1-x}+1\right)}$$

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Problem 80: Result more than twice size of optimal antiderivative.
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$$\int x^6\left(x^7+1\right)^{1/3}dx$$

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Optimal(type 2, 9 leaves, 1 step):
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$$\frac{3\left(x^7+1\right)^{4/3}}{28}$$

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Result(type 2, 36 leaves):
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$$\frac{3(1+x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)\left(x^7+1\right)^{1/3}}{28}$$

Problem 81: Result unnecessarily involves higher level functions.

$$\int \frac{(x^7+1)^{2/3}}{x^8}\,dx$$

Optimal(type 3, 53 leaves, 6 steps):

$$-\frac{(x^7+1)^{2/3}}{7x^7}-\frac{\ln(x)}{3}+\frac{\ln\left(1-(x^7+1)^{1/3}\right)}{7}+\frac{2\arctan\left(\frac{\left(1+2(x^7+1)^{1/3}\right)\sqrt{3}}{3}\right)\sqrt{3}}{21}$$

Result(type 5, 75 leaves):

$$-\frac{(x^7+1)^{2/3}}{7x^7}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+7\ln(x)\right)\pi\sqrt{3}}{3\,\Gamma\left(\frac{2}{3}\right)}-\frac{2\pi\sqrt{3}\,x^7\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^7\right)}{9\,\Gamma\left(\frac{2}{3}\right)}\right)}{21\pi}$$

Problem 82: Unable to integrate problem.

$$\int x^9\sqrt{x^{10}+x^5+1}\,dx$$

Optimal(type 3, 47 leaves, 5 steps):

$$\frac{(x^{10}+x^5+1)^{3/2}}{15}-\frac{3\,\mathrm{arcsinh}\left(\frac{(2x^5+1)\sqrt{3}}{3}\right)}{80}-\frac{(2x^5+1)\sqrt{x^{10}+x^5+1}}{40}$$

Result(type 8, 42 leaves):

$$\frac{(8x^{10}+2x^5+5)\sqrt{x^{10}+x^5+1}}{120}+\int -\frac{3x^4}{16\sqrt{x^{10}+x^5+1}}\,dx$$

Problem 84: Result unnecessarily involves higher level functions.

$$\int \frac{x^3-1}{(x^3+2)^{1/3}}\,dx$$

Optimal(type 3, 48 leaves, 2 steps):

$$\frac{x(x^3+2)^{2/3}}{3}+\frac{5\ln\left(-x+(x^3+2)^{1/3}\right)}{6}-\frac{5\arctan\left(\frac{\left(1+\frac{2x}{(x^3+2)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{9}$$

Result(type 5, 28 leaves):

$$\frac{x\left(x^3+2\right)^{2/3}}{3}-\frac{5\,2^{2/3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-\frac{x^3}{2}\right)}{6}$$

Problem 85: Humongous result has more than 20000 leaves.

$$\int\frac{-x^2+1}{\left(2ax+x^2+1\right)\sqrt{2ax^3+x^4+2bx^2+2ax+1}}\,\mathrm{d}x$$

Optimal(type 3, 66 leaves, 1 step):

$$\frac{\arctan\left(\frac{\left(a+2\left(a^2-b+1\right)x+ax^2\right)\sqrt{2}}{2\sqrt{1-b}\,\sqrt{2ax^3+x^4+2bx^2+2ax+1}}\right)\sqrt{2}}{2\sqrt{1-b}}$$

Result(type ?, 247418 leaves): Display of huge result suppressed!

Problem 89: Result more than twice size of optimal antiderivative.

$$\int\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^4\mathrm{d}x$$

Optimal(type 3, 14 leaves, 3 steps):

$$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{\cos(x)\sin(x)}{8}$$

Result(type 3, 38 leaves):

$$\frac{\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^3+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$$

Problem 98: Result more than twice size of optimal antiderivative.

$$\int\left(1+\cot(x)^3\right)\left(a\sec(x)^2-\sin(2x)\right)^2\mathrm{d}x$$

Optimal(type 3, 80 leaves, 8 steps):

$$\frac{x}{2}+4ax+2\cos(x)^2+\cos(x)^4+4a\cot(x)-\frac{a^2\cot(x)^2}{2}+(4-a)\,a\ln(\cos(x))+\left(a^2+4\right)\ln(\sin(x))+\frac{\cos(x)\sin(x)}{2}-\cos(x)^3\sin(x)+a^2\tan(x)$$
$$+\frac{a^2\tan(x)^3}{3}$$

Result(type 3, 185 leaves):

$$8\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)-4\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)+2\cos(x)^2-4\cot(x)+\cos(x)^4+\frac{x}{2}+2\cos(x)^6+4\ln(\sin(x))+4ax$$

$$+ 2\, a \cot(x)^2 + 4\, a \ln(\sin(x)) - \frac{2\, a^2 \cot(x)}{3} - \frac{a^2}{2 \sin(x)^2} + a^2 \ln(\tan(x)) - 4\, a \ln(\tan(x)) - \frac{2\, a}{\sin(x)^2} + 4\, a \cot(x) + \frac{2 \cos(x)^8}{\sin(x)^2} + \frac{8 \cos(x)^5}{\sin(x)}$$
$$- \frac{2 \cos(x)^6}{\sin(x)^2} - \frac{4 \cos(x)^7}{\sin(x)} + \frac{a^2}{3 \cos(x) \sin(x)} + \frac{a^2}{3 \sin(x) \cos(x)^3}$$

Problem 104: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos(x)^2}{\cos(3\, x)}\, dx$$

Optimal(type 3, 7 leaves, 2 steps):

$$\frac{\operatorname{arctanh}(2 \sin(x))}{2}$$

Result(type 3, 19 leaves):

$$\frac{\ln(2 \sin(x) + 1)}{4} - \frac{\ln(2 \sin(x) - 1)}{4}$$

Problem 107: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{1 + \cos(2\, x)}}\, dx$$

Optimal(type 3, 23 leaves, 2 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sin(2\, x) \sqrt{2}}{2 \sqrt{1 + \cos(2\, x)}}\right) \sqrt{2}}{2}$$

Result(type 5, 8 leaves):

$$\frac{\sqrt{2}\ \mathrm{InverseJacobiAM}(x, 1)}{2}$$

Problem 108: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\cos(x)}{\sqrt{\sin(2\, x)}}\, dx$$

Optimal(type 3, 25 leaves, 1 step):

$$-\frac{\arcsin(\cos(x) - \sin(x))}{2} + \frac{\ln\left(\cos(x) + \sin(x) + \sqrt{\sin(2\, x)}\right)}{2}$$

Result(type 4, 97 leaves):

$$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2-1}}\left(\tan\left(\frac{x}{2}\right)^2-1\right)\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}}$$

Problem 109: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\cos(x)^7}{\sin(2x)^{7/2}}\,dx$$

Optimal(type 3, 47 leaves, 4 steps):

$$-\frac{\arcsin(\cos(x)-\sin(x))}{16}-\frac{\ln\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)}{16}-\frac{\cos(x)^5}{5\sin(2x)^{5/2}}+\frac{\cos(x)}{4\sqrt{\sin(2x)}}$$

Result(type 4, 1107 leaves):

$$\Bigg(\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2-1}}\Bigg(192\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\right.$$

$$\left.\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\tan\left(\frac{x}{2}\right)^6$$

$$-96\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\right.$$

$$\left.\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\tan\left(\frac{x}{2}\right)^6$$

$$-\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\tan\left(\frac{x}{2}\right)^{10}$$

$$-384\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\right.$$

$$\left.\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\tan\left(\frac{x}{2}\right)^4$$

$$+192\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\right.$$

$$\frac{\sqrt{2}}{2}\Bigg)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\tan\left(\frac{x}{2}\right)^4$$

$$+48\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\,\tan\left(\frac{x}{2}\right)^8$$

$$+3\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\tan\left(\frac{x}{2}\right)^8$$

$$+96\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\tan\left(\frac{x}{2}\right)^8$$

$$+192\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticE}\Bigg(\sqrt{1+\tan\left(\frac{x}{2}\right)},$$

$$\frac{\sqrt{2}}{2}\Bigg)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\tan\left(\frac{x}{2}\right)^2$$

$$-96\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\Bigg(\sqrt{1+\tan\left(\frac{x}{2}\right)},$$

$$\frac{\sqrt{2}}{2}\Bigg)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\tan\left(\frac{x}{2}\right)^2$$

$$-144\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\,\tan\left(\frac{x}{2}\right)^6$$

$$+14\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\tan\left(\frac{x}{2}\right)^6$$

$$-192\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\tan\left(\frac{x}{2}\right)^6$$

$$+144\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\,\tan\left(\frac{x}{2}\right)^4$$

$$+14\tan\left(\frac{x}{2}\right)^4\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}$$

$$+96\tan\left(\frac{x}{2}\right)^4\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}$$

$$-48\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\,\tan\left(\frac{x}{2}\right)^2$$

$$+3\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\tan\left(\frac{x}{2}\right)^2$$

$$- \sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\Bigg)\Bigg) \Bigg/ \Bigg(160\tan\left(\frac{x}{2}\right)^3\left(\tan\left(\frac{x}{2}\right)^2\right.$$

$$\left.-1\right)\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\,\left(\tan\left(\frac{x}{2}\right)-1\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)}\,\left(1+\tan\left(\frac{x}{2}\right)\right)\Bigg)$$

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Problem 110: Unable to integrate problem.
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$$\int \frac{1}{\left(\cos(x)^{11}\sin(x)^{13}\right)^{1/4}}\,dx$$

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Optimal(type 3, 60 leaves, 4 steps):
```

$$-\frac{4\cos(x)^5\sin(x)}{9\left(\cos(x)^{11}\sin(x)^{13}\right)^{1/4}}-\frac{8\cos(x)^3\sin(x)^3}{\left(\cos(x)^{11}\sin(x)^{13}\right)^{1/4}}+\frac{4\cos(x)\sin(x)^5}{7\left(\cos(x)^{11}\sin(x)^{13}\right)^{1/4}}$$

```
Result(type 8, 13 leaves):
```

$$\int \frac{1}{\left(\cos(x)^{11}\sin(x)^{13}\right)^{1/4}}\,dx$$

```
Problem 111: Humongous result has more than 20000 leaves.
```

$$\int \frac{-2\sin(2x)+\sqrt{\cos(x)\sin(x)^3}}{-\sqrt{\cos(x)^3\sin(x)}+\sqrt{\tan(x)}}\,dx$$

```
Optimal(type 3, 298 leaves, 66 steps):
```

$$2^{1/4}\operatorname{arccoth}\left(\frac{\cos(x)\left(\sin(x)+\cos(x)\sqrt{2}\right)2^{1/4}}{2\sqrt{\cos(x)^3\sin(x)}}\right)-2^{1/4}\operatorname{arccoth}\left(\frac{\left(\sqrt{2}+\tan(x)\right)2^{1/4}}{2\sqrt{\tan(x)}}\right)+2^{1/4}\arctan\left(\frac{\cos(x)\left(-\sin(x)+\cos(x)\sqrt{2}\right)2^{1/4}}{2\sqrt{\cos(x)^3\sin(x)}}\right)$$

$$-2^{1/4}\arctan\left(\frac{\left(\sqrt{2}-\tan(x)\right)2^{1/4}}{2\sqrt{\tan(x)}}\right)-2\operatorname{arccoth}\left(\frac{\cos(x)\left(\cos(x)+\sin(x)\right)\sqrt{2}}{2\sqrt{\cos(x)^3\sin(x)}}\right)\sqrt{2}-2\arctan\left(\frac{\cos(x)\left(\cos(x)-\sin(x)\right)\sqrt{2}}{2\sqrt{\cos(x)^3\sin(x)}}\right)\sqrt{2}$$

$$+4\csc(x)\sec(x)\sqrt{\cos(x)^3\sin(x)}+\frac{\csc(x)^2\ln\left(1+\cos(x)^2\right)\sec(x)^2\sqrt{\cos(x)^3\sin(x)}\,\sqrt{\cos(x)\sin(x)^3}}{4}$$

$$+\frac{\csc(x)^2\ln(\sin(x))\sec(x)^2\sqrt{\cos(x)^3\sin(x)}\,\sqrt{\cos(x)\sin(x)^3}}{2}+\frac{4}{\sqrt{\tan(x)}}-\frac{\csc(x)^2\ln\left(1+\cos(x)^2\right)\sqrt{\cos(x)\sin(x)^3}\,\sqrt{\tan(x)}}{4}$$

$$+\frac{\csc(x)^2\ln(\sin(x))\sqrt{\cos(x)\sin(x)^3}\,\sqrt{\tan(x)}}{2}$$

```
Result(type ?, 23394 leaves): Display of huge result suppressed!
```

Problem 115: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos(2x)^{3/2}}{\cos(x)^3}\, dx$$

Optimal(type 3, 37 leaves, 6 steps):

$$-\frac{5\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)}{2}+2\arcsin\left(\sin(x)\sqrt{2}\right)\sqrt{2}-\frac{\sec(x)\sqrt{\cos(2x)}\tan(x)}{2}$$

Result(type 3, 99 leaves):

$$\frac{\sqrt{(2\cos(x)^2-1)\sin(x)^2}\left(-4\sqrt{2}\arcsin\left(4\cos(x)^2-3\right)\cos(x)^2+5\arctan\left(\frac{3\cos(x)^2-2}{2\sqrt{-2\sin(x)^4+\sin(x)^2}}\right)\cos(x)^2-2\sqrt{-2\sin(x)^4+\sin(x)^2}\right)}{4\cos(x)^2\sin(x)\sqrt{2\cos(x)^2-1}}$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(4-5\sec(x)^2\right)^{3/2} dx$$

Optimal(type 3, 54 leaves, 7 steps):

$$8\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan(x)^2}}\right)-\frac{7\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-1-5\tan(x)^2}}\right)\sqrt{5}}{2}-\frac{5\sqrt{-1-5\tan(x)^2}\tan(x)}{2}$$

Result(type 4, 753 leaves):

$$\frac{1}{\sqrt{-9-4\sqrt{5}}\left(\sqrt{5}+2\right)(-1+\cos(x))\left(4\cos(x)^2-5\right)^2}\left(-\frac{I}{2}\left(\right.\right.$$

$$-70\,I\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{-9-4\sqrt{5}}\,(-1+\cos(x))}{\sin(x)},-\frac{1}{9+4\sqrt{5}},\right.$$

$$\left.\frac{\sqrt{-9+4\sqrt{5}}}{\sqrt{-9-4\sqrt{5}}}\right)\sin(x)\cos(x)^2\sqrt{5}\sqrt{2}$$

$$+3\,I\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\,\mathrm{EllipticF}\left(\frac{I\left(\sqrt{5}+2\right)(-1+\cos(x))}{\sin(x)},9\right.$$

$$\left.-4\sqrt{5}\right)\sin(x)\cos(x)^2\sqrt{5}\sqrt{2}$$

$$+ 64 I \sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{-9-4\sqrt{5}}\,(-1+\cos(x))}{\sin(x)}, \frac{1}{9+4\sqrt{5}},\right.$$

$$\left.\frac{\sqrt{-9+4\sqrt{5}}}{\sqrt{-9-4\sqrt{5}}}\right)\sin(x)\cos(x)^2\sqrt{5}\sqrt{2}$$

$$- 140 I \sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{-9-4\sqrt{5}}\,(-1+\cos(x))}{\sin(x)},\right.$$

$$\left.-\frac{1}{9+4\sqrt{5}}, \frac{\sqrt{-9+4\sqrt{5}}}{\sqrt{-9-4\sqrt{5}}}\right)\sin(x)\cos(x)^2\sqrt{2}$$

$$+ 6 I \sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\,\mathrm{EllipticF}\left(\frac{I\left(\sqrt{5}+2\right)(-1+\cos(x))}{\sin(x)}, 9\right.$$

$$\left.-4\sqrt{5}\right)\sin(x)\cos(x)^2\sqrt{2}$$

$$+ 128 I \sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{-9-4\sqrt{5}}\,(-1+\cos(x))}{\sin(x)}, \frac{1}{9+4\sqrt{5}},\right.$$

$$\left.\frac{\sqrt{-9+4\sqrt{5}}}{\sqrt{-9-4\sqrt{5}}}\right)\sin(x)\cos(x)^2\sqrt{2} + 80\cos(x)^3\sqrt{5} + 180\cos(x)^3 - 80\cos(x)^2\sqrt{5} - 180\cos(x)^2 - 100\cos(x)\sqrt{5} - 225\cos(x) + 100\sqrt{5} + 225\Bigg)$$

$$\left(\frac{4\cos(x)^2-5}{\cos(x)^2}\right)^{3/2}\sin(x)\cos(x)\Bigg)$$

Problem 117: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-2\cot(x)^2+\sin(x)}{\left(1+5\tan(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 74 leaves, 10 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{2\tan(x)}{\sqrt{1+5\tan(x)^2}}\right)}{4} - \frac{\cos(x)}{4\sqrt{1+5\tan(x)^2}} - \frac{5\cot(x)}{2\sqrt{1+5\tan(x)^2}} - \frac{\cos(x)\sqrt{1+5\tan(x)^2}}{8} + \frac{9\cot(x)\sqrt{1+5\tan(x)^2}}{2}$$

Result(type 4, 974 leaves):

$$\frac{1}{\left(\sqrt{5}+2\right)^{2}\sqrt{-9+4\sqrt{5}}\left(\sqrt{5}-2\right)^{2}\left(4\cos(x)^{2}-5\right)^{2}\sin(x)}\left(\frac{I}{8}\left(-8\,I\,\mathrm{EllipticPi}\left(\frac{\sqrt{-9+4\sqrt{5}}\,(-1+\cos(x))}{\sin(x)},-\frac{1}{-9+4\sqrt{5}},\right.\right.\right.$$

$$\left.\frac{\sqrt{-9-4\sqrt{5}}}{\sqrt{-9+4\sqrt{5}}}\right)\sqrt{2}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sin(x)$$

$$-3\,I\sin(x)\,\mathrm{arctanh}\left(\frac{\sqrt{-16}\cos(x)\,(-1+\cos(x))}{2\sin(x)^{2}\sqrt{-\frac{4\cos(x)^{2}-5}{(1+\cos(x))^{2}}}}\right)\sqrt{5}\sqrt{-\frac{4\cos(x)^{2}-5}{(1+\cos(x))^{2}}}$$

$$+6\,I\sin(x)\cos(x)\,\mathrm{arctanh}\left(\frac{\sqrt{-16}\cos(x)\,(-1+\cos(x))}{2\sin(x)^{2}\sqrt{-\frac{4\cos(x)^{2}-5}{(1+\cos(x))^{2}}}}\right)\sqrt{-\frac{4\cos(x)^{2}-5}{(1+\cos(x))^{2}}}$$

$$+6\,I\,\mathrm{arctanh}\left(\frac{\sqrt{-16}\cos(x)\,(-1+\cos(x))}{2\sin(x)^{2}\sqrt{-\frac{4\cos(x)^{2}-5}{(1+\cos(x))^{2}}}}\right)\sqrt{-\frac{4\cos(x)^{2}-5}{(1+\cos(x))^{2}}}\sin(x)-8\,I\cos(x)\,\mathrm{EllipticPi}\left(\frac{\sqrt{-9+4\sqrt{5}}\,(-1+\cos(x))}{\sin(x)},-\frac{1}{-9+4\sqrt{5}},\right.$$

$$\left.\frac{\sqrt{-9-4\sqrt{5}}}{\sqrt{-9+4\sqrt{5}}}\right)\sqrt{2}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sin(x)$$

$$+4\,I\,\mathrm{EllipticF}\left(\frac{I\left(\sqrt{5}-2\right)(-1+\cos(x))}{\sin(x)},9\right.$$

$$\left.+4\sqrt{5}\right)\sqrt{2}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sin(x)$$

$$+4\,I\cos(x)\,\mathrm{EllipticF}\left(\frac{I\left(\sqrt{5}-2\right)(-1+\cos(x))}{\sin(x)},9\right.$$

$$+4\sqrt{5}\Big)\sqrt{2}\sqrt{\frac{2\cos(x)\sqrt{5}-4\cos(x)-2\sqrt{5}+5}{1+\cos(x)}}\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}+4\cos(x)-5\right)}{1+\cos(x)}}\sin(x)$$

$$-3\cos(x)\arctan\left(\frac{2\cos(x)(-1+\cos(x))}{\sin(x)^2\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)\sqrt{5}\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}\sin(x)$$

$$-3\,\mathrm{I}\sin(x)\cos(x)\,\mathrm{arctanh}\left(\frac{\sqrt{-16}\cos(x)(-1+\cos(x))}{2\sin(x)^2\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)\sqrt{5}\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}+2\sin(x)\cos(x)^2\sqrt{5}$$

$$+6\cos(x)\arctan\left(\frac{2\cos(x)(-1+\cos(x))}{\sin(x)^2\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}\sin(x)$$

$$-3\arctan\left(\frac{2\cos(x)(-1+\cos(x))}{\sin(x)^2\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)\sqrt{5}\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}\sin(x)-4\cos(x)^2\sin(x)$$

$$+6\arctan\left(\frac{2\cos(x)(-1+\cos(x))}{\sin(x)^2\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}\sin(x)-164\cos(x)^2\sqrt{5}-5\sin(x)\sqrt{5}+328\cos(x)^2+10\sin(x)+180\sqrt{5}-360\Bigg)$$

$$\cos(x)^3\left(-\frac{4\cos(x)^2-5}{\cos(x)^2}\right)^{3/2}\Bigg)$$

Problem 119: Result more than twice size of optimal antiderivative.

$$\int\frac{\sec(x)^2-3\sqrt{4\sec(x)^2+5\tan(x)^2}\,\tan(x)}{\sin(x)^2\left(4\sec(x)^2+5\tan(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 45 leaves, 10 steps):

$$-\frac{3\ln(\tan(x))}{4}+\frac{3\ln\left(4+9\tan(x)^2\right)}{8}-\frac{\cot(x)}{4\sqrt{4+9\tan(x)^2}}-\frac{7\tan(x)}{8\sqrt{4+9\tan(x)^2}}$$

Result(type 3, 116 leaves):

$$-\frac{1}{8\sin(x)\cos(x)^3\left(-\frac{5\cos(x)^2-9}{\cos(x)^2}\right)^{3/2}}\left(6\sin(x)\cos(x)^3\left(-\frac{5\cos(x)^2-9}{\cos(x)^2}\right)^{3/2}\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)-3\sin(x)\cos(x)^3\left(-\frac{5\cos(x)^2-9}{\cos(x)^2}\right)^{3/2}\ln\left(-\frac{5\cos(x)^2-9}{(1+\cos(x))^2}\right)+25\cos(x)^4-80\cos(x)^2+63\right)$$

```
Problem 120: Unable to integrate problem.
```

$$\int\frac{\cot(x)}{(a^4+b^4\csc(x)^2)^{1/4}}\,dx$$

```
Optimal(type 3, 48 leaves, 6 steps):
```

$$-\frac{\arctan\left(\frac{(a^4+b^4\csc(x)^2)^{1/4}}{a}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{(a^4+b^4\csc(x)^2)^{1/4}}{a}\right)}{a}$$

```
Result(type 8, 19 leaves):
```

$$\int\frac{\cot(x)}{(a^4+b^4\csc(x)^2)^{1/4}}\,dx$$

```
Problem 121: Result more than twice size of optimal antiderivative.
```

$$\int\frac{-\cos(2x)+2\tan(x)^2}{\cos(x)^2(\tan(x)\tan(2x))^{3/2}}\,dx$$

```
Optimal(type 3, 78 leaves, ? steps):
```

$$2\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)-\frac{11\operatorname{arctanh}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)\sqrt{2}}{8}+\frac{3\tan(x)}{4\sqrt{\tan(x)\tan(2x)}}+\frac{\tan(x)}{2(\tan(x)\tan(2x))^{3/2}}+\frac{2\tan(x)^3}{3(\tan(x)\tan(2x))^{3/2}}$$

```
Result(type 3, 558 leaves):
```

$$\frac{1}{96\sin(x)^3\cos(x)^3\left(\frac{\sin(x)^2}{2\cos(x)^2-1}\right)^{3/2}\left(\frac{2\cos(x)^2-1}{(1+\cos(x))^2}\right)^{3/2}}\left(\sqrt{2}\sqrt{4}(-1+\cos(x))^2\left(48\cos(x)^4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)\sqrt{4}(-1+\cos(x))}{2\sin(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}\right)\right.\right.$$

$$-33\cos(x)^4\operatorname{arctanh}\left(\frac{\sqrt{4}(2\cos(x)^2-3\cos(x)+1)}{2\sin(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}\right)+168\cos(x)^4\ln\Bigg($$

$$-\frac{4\left(\cos(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}-2\cos(x)^2+\cos(x)-\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}+1\right)}{\sin(x)^2}\Bigg)-201\cos(x)^4\ln\Bigg($$

$$-\frac{2\left(\cos(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}-2\cos(x)^2+\cos(x)-\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}+1\right)}{\sin(x)^2}\Bigg)-22\cos(x)^4\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}$$

$$-48\cos(x)^3\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\cos(x)\sqrt{4}\,(-1+\cos(x))}{2\sin(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}\right)+33\cos(x)^3\,\mathrm{arctanh}\left(\frac{\sqrt{4}\,(2\cos(x)^2-3\cos(x)+1)}{2\sin(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}\right)-168\cos(x)^3\ln\Bigg($$

$$-\frac{4\left(\cos(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}-2\cos(x)^2+\cos(x)-\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}+1\right)}{\sin(x)^2}\Bigg)+201\cos(x)^3\ln\Bigg($$

$$-\frac{2\left(\cos(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}-2\cos(x)^2+\cos(x)-\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}+1\right)}{\sin(x)^2}\Bigg)+36\cos(x)^2\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}-8\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}\Bigg)\Bigg)$$

```
Problem 122: Unable to integrate problem.
```

$$\int \frac{\cot(x)\left(1+\left(1-8\tan(x)^2\right)^{1/3}\right)}{\cos(x)^2\left(1-8\tan(x)^2\right)^{2/3}}\,dx$$

```
Optimal(type 3, 23 leaves, 15 steps):
```

$$-\ln(\tan(x))+\frac{3\ln\left(1-\left(1-8\tan(x)^2\right)^{1/3}\right)}{2}$$

```
Result(type 8, 31 leaves):
```

$$\int \frac{\cot(x)\left(1+\left(1-8\tan(x)^2\right)^{1/3}\right)}{\cos(x)^2\left(1-8\tan(x)^2\right)^{2/3}}\,dx$$

```
Problem 123: Unable to integrate problem.
```

$$\int \frac{\left(5\cos(x)^2-\sqrt{-1+5\sin(x)^2}\right)\tan(x)}{\left(-1+5\sin(x)^2\right)^{1/4}\left(2+\sqrt{-1+5\sin(x)^2}\right)}\,dx$$

Optimal(type 3, 81 leaves, 14 steps):

$$2\left(-1+5\sin(x)^2\right)^{1/4}-\frac{3\arctan\left(\frac{\left(-1+5\sin(x)^2\right)^{1/4}\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{\left(-1+5\sin(x)^2\right)^{1/4}\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{\left(-1+5\sin(x)^2\right)^{1/4}}{2\left(2+\sqrt{-1+5\sin(x)^2}\right)}$$

Result(type 8, 48 leaves):

$$\int\frac{\left(5\cos(x)^2-\sqrt{-1+5\sin(x)^2}\right)\tan(x)}{\left(-1+5\sin(x)^2\right)^{1/4}\left(2+\sqrt{-1+5\sin(x)^2}\right)}\,dx$$

Problem 139: Unable to integrate problem.

$$\int\left(1+a^{mx}\right)^n\,dx$$

Optimal(type 5, 42 leaves, 2 steps):

$$-\frac{\left(1+a^{mx}\right)^{1+n}\operatorname{hypergeom}\left([1,1+n],[2+n],1+a^{mx}\right)}{m\,(1+n)\,\ln(a)}$$

Result(type 8, 11 leaves):

$$\int\left(1+a^{mx}\right)^n\,dx$$

Problem 147: Unable to integrate problem.

$$\int\frac{e^x}{1-\cos(x)}\,dx$$

Optimal(type 5, 25 leaves, 2 steps):

$$(-1+I)\,e^{(1+I)\,x}\operatorname{hypergeom}\left([2,1-I],[2-I],e^{I\,x}\right)$$

Result(type 8, 31 leaves):

$$-\frac{2\,I\,e^x}{e^{I\,x}-1}+\int\frac{2\,I\,e^x}{e^{I\,x}-1}\,dx$$

Problem 148: Result more than twice size of optimal antiderivative.

$$\int\frac{e^x\,(1-\sin(x))}{1-\cos(x)}\,dx$$

Optimal(type 3, 14 leaves, 1 step):

$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

Result(type 3, 32 leaves):

$$\frac{-e^x \tan\left(\frac{x}{2}\right)^2 - e^x}{\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)\tan\left(\frac{x}{2}\right)}$$

Problem 149: Unable to integrate problem.

$$\int \frac{e^x (1 + \sin(x))}{1 - \cos(x)}\, dx$$

Optimal(type 5, 39 leaves, 7 steps):

$$(-2 + 2\,I)\, e^{(1 + I)\, x} \mathrm{hypergeom}([2, 1 - I], [2 - I], e^{I x}) + \frac{e^x \sin(x)}{1 - \cos(x)}$$

Result(type 8, 36 leaves):

$$I\, e^x - \frac{2\, I\, e^x}{e^{I x} - 1} + \int \frac{4\, I\, e^x}{e^{I x} - 1}\, dx$$

Problem 151: Unable to integrate problem.

$$\int \frac{e^x (1 - \cos(x))}{1 - \sin(x)}\, dx$$

Optimal(type 5, 43 leaves, 7 steps):

$$(2 + 2\,I)\, e^{(1 + I)\, x} \mathrm{hypergeom}([2, 1 - I], [2 - I], -I\, e^{I x}) - \frac{e^x \cos(x)}{1 - \sin(x)}$$

Result(type 8, 36 leaves):

$$I\, e^x + \frac{2\, e^x}{e^{I x} - I} + \int -\frac{4\, e^x}{e^{I x} - I}\, dx$$

Problem 152: Result more than twice size of optimal antiderivative.

$$\int \frac{e^x (1 + \cos(x))}{1 - \sin(x)}\, dx$$

Optimal(type 3, 13 leaves, 1 step):

$$\frac{e^x \cos(x)}{1 - \sin(x)}$$

Result(type 3, 52 leaves):

$$\frac{-e^x \tan\left(\frac{x}{2}\right) - e^x \tan\left(\frac{x}{2}\right)^2 - e^x \tan\left(\frac{x}{2}\right)^3 - e^x}{\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)\left(\tan\left(\frac{x}{2}\right) - 1\right)}$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{e^{2x}\cosh(x)^4}\, dx$$

Optimal(type 3, 10 leaves, 3 steps):

$$-\frac{8}{3\left(1+e^{2x}\right)^3}$$

Result(type 3, 51 leaves):

$$-\frac{2\left(-\tanh\left(\frac{x}{2}\right)^5+2\tanh\left(\frac{x}{2}\right)^4-\frac{10\tanh\left(\frac{x}{2}\right)^3}{3}+2\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^3}$$

Problem 171: Unable to integrate problem.

$$\int \frac{\ln(\ln(x))^n}{x}\, dx$$

Optimal(type 4, 24 leaves, 3 steps):

$$\frac{\Gamma(1+n, -\ln(\ln(x)))\ln(\ln(x))^n}{(-\ln(\ln(x)))^n}$$

Result(type 8, 11 leaves):

$$\int \frac{\ln(\ln(x))^n}{x}\, dx$$

Problem 173: Result more than twice size of optimal antiderivative.

$$\int \frac{\ln\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\, dx$$

Optimal(type 3, 22 leaves, 4 steps):

$$-\frac{x}{2}+\frac{\ln\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)}+\tan\left(\frac{x}{2}\right)$$

Result(type 3, 163 leaves):

$$-\frac{2 I\ln\left(e^{\frac{I}{2}x}\right)}{e^{Ix}+1}+\frac{1}{e^{Ix}+1}\left(\pi\,\mathrm{csgn}\left(I\left(e^{Ix}+1\right)\right)\mathrm{csgn}\left(I e^{-\frac{I}{2}x}\right)\mathrm{csgn}\left(I\cos\left(\frac{x}{2}\right)\right)-\pi\,\mathrm{csgn}\left(I\left(e^{Ix}+1\right)\right)\mathrm{csgn}\left(I\cos\left(\frac{x}{2}\right)\right)^2\right.$$

$$-\pi\,\mathrm{csgn}\left(\mathrm{I}\,e^{-\frac{\mathrm{I}}{2}x}\right)\mathrm{csgn}\left(\mathrm{I}\cos\left(\frac{x}{2}\right)\right)^2+\pi\,\mathrm{csgn}\left(\mathrm{I}\cos\left(\frac{x}{2}\right)\right)^3-\mathrm{I}\ln\left(e^{\mathrm{I}x}+1\right)e^{\mathrm{I}x}-x\,e^{\mathrm{I}x}-2\,\mathrm{I}\ln(2)+\mathrm{I}\ln\left(e^{\mathrm{I}x}+1\right)+2\,\mathrm{I}-x\Big)$$

```
Problem 175: Result more than twice size of optimal antiderivative.
```

$$\int x^3\left(-x^2+1\right)^{3/2}\arccos(x)\,dx$$

```
Optimal(type 3, 45 leaves, 4 steps):
```

$$-\frac{2x}{35}-\frac{x^3}{105}+\frac{8x^5}{175}-\frac{x^7}{49}-\frac{\left(-x^2+1\right)^{5/2}\arccos(x)}{5}+\frac{\left(-x^2+1\right)^{7/2}\arccos(x)}{7}$$

```
Result(type 3, 429 leaves):
```

$$\frac{(\mathrm{I}+7\arccos(x))\left(64\,\mathrm{I}x^7-64\sqrt{-x^2+1}\,x^6-112\,\mathrm{I}x^5+80\sqrt{-x^2+1}\,x^4+56\,\mathrm{I}x^3-24x^2\sqrt{-x^2+1}-7\,\mathrm{I}x+\sqrt{-x^2+1}\right)}{6272}$$
$$-\frac{(\mathrm{I}+5\arccos(x))\left(16\,\mathrm{I}x^5-16\sqrt{-x^2+1}\,x^4-20\,\mathrm{I}x^3+12x^2\sqrt{-x^2+1}+5\,\mathrm{I}x-\sqrt{-x^2+1}\right)}{3200}$$
$$-\frac{(\mathrm{I}+3\arccos(x))\left(4\,\mathrm{I}x^3-4x^2\sqrt{-x^2+1}-3\,\mathrm{I}x+\sqrt{-x^2+1}\right)}{384}+\frac{3\,(\arccos(x)+\mathrm{I})\left(\mathrm{I}x-\sqrt{-x^2+1}\right)}{128}-\frac{3\,(\arccos(x)-\mathrm{I})\left(\mathrm{I}x+\sqrt{-x^2+1}\right)}{128}$$
$$+\frac{(-\mathrm{I}+3\arccos(x))\left(4\,\mathrm{I}x^3+4x^2\sqrt{-x^2+1}-3\,\mathrm{I}x-\sqrt{-x^2+1}\right)}{384}$$
$$+\frac{(-\mathrm{I}+5\arccos(x))\left(16\,\mathrm{I}x^5+16\sqrt{-x^2+1}\,x^4-20\,\mathrm{I}x^3-12x^2\sqrt{-x^2+1}+5\,\mathrm{I}x+\sqrt{-x^2+1}\right)}{3200}$$
$$-\frac{(-\mathrm{I}+7\arccos(x))\left(64\,\mathrm{I}x^7+64\sqrt{-x^2+1}\,x^6-112\,\mathrm{I}x^5-80\sqrt{-x^2+1}\,x^4+56\,\mathrm{I}x^3+24x^2\sqrt{-x^2+1}-7\,\mathrm{I}x-\sqrt{-x^2+1}\right)}{6272}$$

```
Problem 176: Result more than twice size of optimal antiderivative.
```

$$\int\frac{\left(-x^2+1\right)^{3/2}\arccos(x)}{x}\,dx$$

```
Optimal(type 4, 98 leaves, 10 steps):
```

$$\frac{4x}{3}-\frac{x^3}{9}+\frac{\left(-x^2+1\right)^{3/2}\arccos(x)}{3}+2\,\mathrm{I}\arccos(x)\arctan\left(x+\mathrm{I}\sqrt{-x^2+1}\right)-\mathrm{I}\,\mathrm{polylog}\left(2,-\mathrm{I}\left(x+\mathrm{I}\sqrt{-x^2+1}\right)\right)+\mathrm{I}\,\mathrm{polylog}\left(2,\mathrm{I}\left(x+\mathrm{I}\sqrt{-x^2+1}\right)\right)$$
$$+\arccos(x)\sqrt{-x^2+1}$$

```
Result(type 4, 226 leaves):
```

$$\frac{(\mathrm{I}+3\arccos(x))\left(4\,\mathrm{I}x^3-4x^2\sqrt{-x^2+1}-3\,\mathrm{I}x+\sqrt{-x^2+1}\right)}{72}-\frac{5\,(\arccos(x)+\mathrm{I})\left(\mathrm{I}x-\sqrt{-x^2+1}\right)}{8}+\frac{5\,(\arccos(x)-\mathrm{I})\left(\mathrm{I}x+\sqrt{-x^2+1}\right)}{8}$$

$$-\frac{(-I+3\arccos(x))\left(4 I x^3+4 x^2\sqrt{-x^2+1}-3 I x-\sqrt{-x^2+1}\right)}{72}+\ln\left(1+I\left(x+I\sqrt{-x^2+1}\right)\right)\arccos(x)-\ln\left(1-I\left(x+I\sqrt{-x^2+1}\right)\right)\arccos(x)$$
$$-I\,\mathrm{dilog}\left(1+I\left(x+I\sqrt{-x^2+1}\right)\right)+I\,\mathrm{dilog}\left(1-I\left(x+I\sqrt{-x^2+1}\right)\right)$$

Problem 177: Result more than twice size of optimal antiderivative.

$$\int\frac{x\arccos(x)}{(-x^2+1)^{3/2}}\,dx$$

Optimal(type 3, 15 leaves, 2 steps):

$$\mathrm{arctanh}(x)+\frac{\arccos(x)}{\sqrt{-x^2+1}}$$

Result(type 3, 46 leaves):

$$-\frac{\sqrt{-x^2+1}\arccos(x)}{x^2-1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}-\frac{x}{\sqrt{-x^2+1}}\right)$$

Problem 179: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3\arcsin(x)}{(-x^2+1)^{3/2}}\,dx$$

Optimal(type 3, 32 leaves, 3 steps):

$$-x-\mathrm{arctanh}(x)+\frac{\arcsin(x)}{\sqrt{-x^2+1}}+\arcsin(x)\sqrt{-x^2+1}$$

Result(type 3, 101 leaves):

$$\frac{(\arcsin(x)+I)\left(I x+\sqrt{-x^2+1}\right)}{2}-\frac{\left(I x-\sqrt{-x^2+1}\right)(\arcsin(x)-I)}{2}-\frac{\sqrt{-x^2+1}\arcsin(x)}{x^2-1}-\ln\left(I x+\sqrt{-x^2+1}+I\right)+\ln\left(I x+\sqrt{-x^2+1}-I\right)$$

Problem 185: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{arcsec}(x)}{(x^2-1)^{5/2}}\,dx$$

Optimal(type 3, 49 leaves, 4 steps):

$$\frac{5\,\mathrm{arccoth}\left(\sqrt{x^2}\right)}{6}-\frac{x\,\mathrm{arcsec}(x)}{3\,(x^2-1)^{3/2}}+\frac{\sqrt{x^2}}{6\,(-x^2+1)}+\frac{2\,x\,\mathrm{arcsec}(x)}{3\sqrt{x^2-1}}$$

Result(type 3, 127 leaves):

$$\frac{\sqrt{x^2-1}\,x\left(4\,\mathrm{arcsec}(x)\,x^2-\sqrt{\frac{x^2-1}{x^2}}\,x-6\,\mathrm{arcsec}(x)\right)}{6\left(x^4-2x^2+1\right)}+\frac{5\sqrt{\frac{x^2-1}{x^2}}\,x\ln\left(\frac{1}{x}+I\sqrt{1-\frac{1}{x^2}}+1\right)}{6\sqrt{x^2-1}}-\frac{5\sqrt{\frac{x^2-1}{x^2}}\,x\ln\left(\frac{1}{x}+I\sqrt{1-\frac{1}{x^2}}-1\right)}{6\sqrt{x^2-1}}$$

Problem 186: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{arccsc}(x)}{x^2\left(x^2-1\right)^{5/2}}\,dx$$

Optimal(type 3, 56 leaves, 5 steps):

$$-\frac{11\,\mathrm{arccoth}\left(\sqrt{x^2}\right)}{6}+\frac{\left(8x^4-12x^2+3\right)\mathrm{arccsc}(x)}{3x\left(x^2-1\right)^{3/2}}-\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2}}{6\left(x^2-1\right)}$$

Result(type 3, 202 leaves):

$$\frac{\left(I\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)\left(\mathrm{arccsc}(x)+I\right)}{2\sqrt{x^2-1}\,x}+\frac{\left(-I\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)\left(\mathrm{arccsc}(x)-I\right)}{2\sqrt{x^2-1}\,x}+\frac{\sqrt{x^2-1}\,x\left(10\,\mathrm{arccsc}(x)\,x^2+\sqrt{\frac{x^2-1}{x^2}}\,x-12\,\mathrm{arccsc}(x)\right)}{6\left(x^4-2x^2+1\right)}$$
$$-\frac{11\sqrt{\frac{x^2-1}{x^2}}\,x\ln\left(\frac{I}{x}+\sqrt{1-\frac{1}{x^2}}+I\right)}{6\sqrt{x^2-1}}+\frac{11\sqrt{\frac{x^2-1}{x^2}}\,x\ln\left(\frac{I}{x}+\sqrt{1-\frac{1}{x^2}}-I\right)}{6\sqrt{x^2-1}}$$

Problem 187: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(x^2-1\right)^{3/2}\mathrm{arcsec}(x)^2}{x^5}\,dx$$

Optimal(type 3, 105 leaves, 11 steps):

$$-\frac{\left(x^2-1\right)^{3/2}\mathrm{arcsec}(x)^2}{4x^4}-\frac{3\,\mathrm{arcsec}(x)}{8x\sqrt{x^2}}+\frac{9x\,\mathrm{arcsec}(x)}{64\sqrt{x^2}}+\frac{\left(x^2-1\right)^2\mathrm{arcsec}(x)}{8x^3\sqrt{x^2}}+\frac{x\,\mathrm{arcsec}(x)^3}{8\sqrt{x^2}}+\frac{\left(17x^2-2\right)\sqrt{x^2-1}}{64x^4}-\frac{3\,\mathrm{arcsec}(x)^2\sqrt{x^2-1}}{8x^2}$$

Result(type 3, 326 leaves):

$$\frac{\sqrt{\frac{x^2-1}{x^2}}\,x\,\mathrm{arcsec}(x)^3}{8\sqrt{x^2-1}}-\frac{\left(I\sqrt{\frac{x^2-1}{x^2}}\,x^5-8I\sqrt{\frac{x^2-1}{x^2}}\,x^3+4x^4+8I\sqrt{\frac{x^2-1}{x^2}}\,x-12x^2+8\right)\left(4I\,\mathrm{arcsec}(x)+8\,\mathrm{arcsec}(x)^2-1\right)}{512\sqrt{x^2-1}\,x^4}$$
$$-\frac{\left(I\sqrt{\frac{x^2-1}{x^2}}\,x^3-2I\sqrt{\frac{x^2-1}{x^2}}\,x+2x^2-2\right)\left(2\,\mathrm{arcsec}(x)^2-1+2I\,\mathrm{arcsec}(x)\right)}{16\sqrt{x^2-1}\,x^2}$$

$$+\frac{\left(I\sqrt{\frac{x^2-1}{x^2}}\,x^3-2\,I\sqrt{\frac{x^2-1}{x^2}}\,x-2\,x^2+2\right)\left(2\,\mathrm{arcsec}(x)^2-1-2\,I\,\mathrm{arcsec}(x)\right)}{16\sqrt{x^2-1}\,x^2}$$

$$+\frac{\left(I\sqrt{\frac{x^2-1}{x^2}}\,x^5-8\,I\sqrt{\frac{x^2-1}{x^2}}\,x^3-4\,x^4+8\,I\sqrt{\frac{x^2-1}{x^2}}\,x+12\,x^2-8\right)\left(-4\,I\,\mathrm{arcsec}(x)+8\,\mathrm{arcsec}(x)^2-1\right)}{512\sqrt{x^2-1}\,x^4}$$

Problem 188: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{arcsec}(x)^3\sqrt{x^2-1}}{x^4}\,dx$$

Optimal(type 3, 92 leaves, 8 steps):

$$-\frac{2\,(x^2-1)^{3/2}\,\mathrm{arcsec}(x)}{9\,x^3}+\frac{(x^2-1)^{3/2}\,\mathrm{arcsec}(x)^3}{3\,x^3}+\frac{2\,(-21\,x^2+1)}{27\,x^2\sqrt{x^2}}+\frac{2\,\mathrm{arcsec}(x)^2}{3\sqrt{x^2}}+\frac{(x^2-1)\,\mathrm{arcsec}(x)^2}{3\,x^2\sqrt{x^2}}-\frac{4\,\mathrm{arcsec}(x)\sqrt{x^2-1}}{3\,x}$$

Result(type 3, 249 leaves):

$$\frac{\left(-5\,x^2+4-3\,I\sqrt{\frac{x^2-1}{x^2}}\,x^3+x^4+4\,I\sqrt{\frac{x^2-1}{x^2}}\,x\right)\left(9\,I\,\mathrm{arcsec}(x)^2+9\,\mathrm{arcsec}(x)^3-2\,I-6\,\mathrm{arcsec}(x)\right)}{216\sqrt{x^2-1}\,x^3}$$

$$+\frac{\left(-I\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)\left(\mathrm{arcsec}(x)^3-6\,\mathrm{arcsec}(x)+3\,I\,\mathrm{arcsec}(x)^2-6\,I\right)}{8\sqrt{x^2-1}\,x}$$

$$+\frac{\left(I\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)\left(\mathrm{arcsec}(x)^3-6\,\mathrm{arcsec}(x)-3\,I\,\mathrm{arcsec}(x)^2+6\,I\right)}{8\sqrt{x^2-1}\,x}$$

$$+\frac{\left(3\,I\sqrt{\frac{x^2-1}{x^2}}\,x^3+x^4-4\,I\sqrt{\frac{x^2-1}{x^2}}\,x-5\,x^2+4\right)\left(-9\,I\,\mathrm{arcsec}(x)^2+9\,\mathrm{arcsec}(x)^3+2\,I-6\,\mathrm{arcsec}(x)\right)}{216\sqrt{x^2-1}\,x^3}$$

Problem 193: Unable to integrate problem.

$$\int\arcsin(\sinh(x))\,\mathrm{sech}(x)^4\,dx$$

Optimal(type 3, 40 leaves, 5 steps):

$$-\frac{2\arcsin\left(\frac{\cosh(x)\sqrt{2}}{2}\right)}{3}+\frac{\operatorname{sech}(x)\sqrt{1-\sinh(x)^2}}{6}+\arcsin(\sinh(x))\tanh(x)-\frac{\arcsin(\sinh(x))\tanh(x)^3}{3}$$

Result(type 8, 10 leaves):

$$\int \arcsin(\sinh(x))\operatorname{sech}(x)^4\,dx$$

Test results for the 32 problems in "Welz Problems.txt"

Problem 2: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{b^2x^2+2a^2+b^2}}{-b\left(2a^2+b^2\right)+4a\left(2a^2+b^2\right)x-b^3x^2+8a\left(a^2+b^2\right)x^3+b\left(2a^2+b^2\right)x^4+4ab^2x^5+b^3x^6}\,dx$$

Optimal(type 1, 1 leaves, 0 steps):

$$0$$

Result(type ?, 8793 leaves): Display of huge result suppressed!

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x^2-1}}{(-I+x)^2}\,dx$$

Optimal(type 3, 52 leaves, 6 steps):

$$\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{I\arctan\left(\frac{(1-Ix)\sqrt{2}}{2\sqrt{x^2-1}}\right)\sqrt{2}}{2}+\frac{\sqrt{x^2-1}}{I-x}$$

Result(type 3, 124 leaves):

$$\frac{\left((-I+x)^2+2I(-I+x)-2\right)^{3/2}}{2(-I+x)}+\ln\left(x+\sqrt{(-I+x)^2+2I(-I+x)-2}\right)+\frac{I\arctan\left(\frac{(-4+2I(-I+x))\sqrt{2}}{4\sqrt{(-I+x)^2+2I(-I+x)-2}}\right)\sqrt{2}}{2}$$
$$-\frac{I\sqrt{(-I+x)^2+2I(-I+x)-2}}{2}-\frac{x\sqrt{(-I+x)^2+2I(-I+x)-2}}{2}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

Optimal(type 3, 158 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{x^2-1}\right)} - \frac{\arctan\left(\frac{\sqrt{x^2-1}\,\sqrt{-2+2\sqrt{5}}}{2-x\left(-\sqrt{5}+1\right)}\right)\sqrt{-110+50\sqrt{5}}}{50} + \frac{\arctan\left(\frac{\sqrt{x}\,\sqrt{2+2\sqrt{5}}}{2}\right)\sqrt{-110+50\sqrt{5}}}{25}$$

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{x}\,\sqrt{-2+2\sqrt{5}}}{2}\right)\sqrt{110+50\sqrt{5}}}{25} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2-1}\,\sqrt{2+2\sqrt{5}}}{2-x-x\sqrt{5}}\right)\sqrt{110+50\sqrt{5}}}{50}$$

Result(type 3, 901 leaves):

$$- \frac{6\sqrt{5}\,\operatorname{arctanh}\left(\frac{2\left(1+\sqrt{5}+\left(\sqrt{5}+1\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{\sqrt{2+2\sqrt{5}}\,\sqrt{4\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+4\left(\sqrt{5}+1\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+2+2\sqrt{5}}}\right)}{25\sqrt{2+2\sqrt{5}}}$$

$$- \frac{6\sqrt{5}\,\arctan\left(\frac{2\left(1-\sqrt{5}+\left(-\sqrt{5}+1\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{\sqrt{-2+2\sqrt{5}}\,\sqrt{4\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+4\left(-\sqrt{5}+1\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+2-2\sqrt{5}}}\right)}{25\sqrt{-2+2\sqrt{5}}}$$

$$- \frac{\sqrt{\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+\left(-\sqrt{5}+1\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+\frac{1}{2}-\frac{\sqrt{5}}{2}}}{5\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}$$

$$+ \frac{2\arctan\left(\frac{2\left(1-\sqrt{5}+\left(-\sqrt{5}+1\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{\sqrt{-2+2\sqrt{5}}\,\sqrt{4\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+4\left(-\sqrt{5}+1\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+2-2\sqrt{5}}}\right)\sqrt{5}}{5\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{-2+2\sqrt{5}}}$$

$$
-\frac{6\arctan\left(\frac{2\left(1-\sqrt{5}+\left(-\sqrt{5}+1\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{\sqrt{-2+2\sqrt{5}}\,\sqrt{4\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+4\left(-\sqrt{5}+1\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+2-2\sqrt{5}}}\right)}{5\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{-2+2\sqrt{5}}}
$$

$$
+\frac{\sqrt{5}\sqrt{\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+\left(-\sqrt{5}+1\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+\frac{1}{2}-\frac{\sqrt{5}}{2}}}{5\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}-\frac{\sqrt{\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+\left(\sqrt{5}+1\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+\frac{1}{2}+\frac{\sqrt{5}}{2}}}{5\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}
$$

$$
+\frac{6\,\mathrm{arctanh}\left(\frac{2\left(1+\sqrt{5}+\left(\sqrt{5}+1\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{\sqrt{2+2\sqrt{5}}\,\sqrt{4\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+4\left(\sqrt{5}+1\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+2+2\sqrt{5}}}\right)}{5\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{2+2\sqrt{5}}}
$$

$$
+\frac{2\,\mathrm{arctanh}\left(\frac{2\left(1+\sqrt{5}+\left(\sqrt{5}+1\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{\sqrt{2+2\sqrt{5}}\,\sqrt{4\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+4\left(\sqrt{5}+1\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+2+2\sqrt{5}}}\right)\sqrt{5}}{5\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{2+2\sqrt{5}}}
$$

$$
-\frac{\sqrt{5}\sqrt{\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2+\left(\sqrt{5}+1\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+\frac{1}{2}+\frac{\sqrt{5}}{2}}}{5\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{2\sqrt{x}}{5\left(x-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}-\frac{4\,\mathrm{arctanh}\left(\frac{2\sqrt{x}}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}
$$

$$
-\frac{8\,\mathrm{arctanh}\left(\frac{2\sqrt{x}}{\sqrt{2+2\sqrt{5}}}\right)\sqrt{5}}{25\sqrt{2+2\sqrt{5}}}+\frac{2\sqrt{x}}{5\left(x+\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{4\arctan\left(\frac{2\sqrt{x}}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}}-\frac{8\arctan\left(\frac{2\sqrt{x}}{\sqrt{-2+2\sqrt{5}}}\right)\sqrt{5}}{25\sqrt{-2+2\sqrt{5}}}
$$

```
Problem 5: Unable to integrate problem.
```

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\, dx$$

Optimal(type 3, 24 leaves, 2 steps):

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2+\sqrt{x^4+1}}}\right)\sqrt{2}}{2}$$

Result(type 8, 23 leaves):

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\, dx$$

Problem 6: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\, dx$$

Optimal(type 3, 26 leaves, 2 steps):

$$\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{-x^2+\sqrt{x^4+1}}}\right)\sqrt{2}}{2}$$

Result(type 5, 21 leaves):

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \left(x+\sqrt{x^2+a}\right)^b dx$$

Optimal(type 3, 44 leaves, 3 steps):

$$-\frac{a\left(x+\sqrt{x^2+a}\right)^{-1+b}}{2\,(1-b)}+\frac{\left(x+\sqrt{x^2+a}\right)^{1+b}}{2\,(1+b)}$$

Result(type 3, 119 leaves):

$$\frac{a^{\frac{b}{2}+\frac{1}{2}}\, b \left(\frac{8\sqrt{\pi}\, x^{1+b}\, a^{-\frac{b}{2}-\frac{1}{2}} \left(\frac{b\, a}{x^2}+b-1\right) \left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+b}}{(1+b)\, b\, (-2+2\, b)} + \frac{4\sqrt{\pi}\, x^{1+b}\, a^{-\frac{b}{2}-\frac{1}{2}} \sqrt{\frac{a}{x^2}+1} \left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+b}}{(1+b)\, b}\right)}{4\sqrt{\pi}}$$

```
Problem 9: Unable to integrate problem.
```

$$\int \left(x-\sqrt{x^2+a}\right)^b \,\mathrm{d}x$$

```
Optimal(type 3, 48 leaves, 3 steps):
```

$$-\frac{a\left(x-\sqrt{x^2+a}\right)^{-1+b}}{2\,(1-b)} + \frac{\left(x-\sqrt{x^2+a}\right)^{1+b}}{2\,(1+b)}$$

```
Result(type 8, 15 leaves):
```

$$\int \left(x-\sqrt{x^2+a}\right)^b \,\mathrm{d}x$$

```
Problem 11: Result unnecessarily involves higher level functions.
```

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \,\mathrm{d}x$$

```
Optimal(type 3, 62 leaves, 6 steps):
```

$$-2\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)\sqrt{a} - 2\,\mathrm{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)\sqrt{a} + 2\sqrt{x+\sqrt{a^2+x^2}}$$

```
Result(type 5, 24 leaves):
```

$$2\sqrt{2}\,\sqrt{x}\;\mathrm{hypergeom}\left(\left[-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}\right], \left[\frac{1}{2}, \frac{3}{4}\right], -\frac{a^2}{x^2}\right)$$

```
Problem 12: Result unnecessarily involves higher level functions.
```

$$\int \frac{1}{x\,(-x^2+1)^{2/3}} \,\mathrm{d}x$$

```
Optimal(type 3, 45 leaves, 5 steps):
```

$$-\frac{\ln(x)}{2} + \frac{3\ln\left(1-(-x^2+1)^{1/3}\right)}{4} - \frac{\arctan\left(\frac{\left(1+2\,(-x^2+1)^{1/3}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}$$

```
Result(type 5, 47 leaves):
```

$$\frac{\left(\frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + 2\ln(x) + I\pi\right)\Gamma\left(\frac{2}{3}\right) + \frac{2\Gamma\left(\frac{2}{3}\right)x^2 \operatorname{hypergeom}\left(\left[1, 1, \frac{5}{3}\right], [2, 2], x^2\right)}{3}}{2\Gamma\left(\frac{2}{3}\right)}$$

Problem 13: Unable to integrate problem.

$$\int \frac{x}{(1+x)\,(-x^3+1)^{1/3}}\,dx$$

Optimal(type 3, 113 leaves, 3 steps):

$$\frac{\ln\left((1-x)\,(1+x)^2\right)2^{2/3}}{8} + \frac{\ln\left(x+(-x^3+1)^{1/3}\right)}{2} - \frac{3\ln\left(-1+x+2^{2/3}\,(-x^3+1)^{1/3}\right)2^{2/3}}{8} - \frac{\arctan\left(\frac{\left(1-\frac{2x}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3} + \frac{\arctan\left(\frac{\left(1+\frac{2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}\,2^{2/3}}{4}$$

Result(type 8, 18 leaves):

$$\int \frac{x}{(1+x)\,(-x^3+1)^{1/3}}\,dx$$

Problem 14: Unable to integrate problem.

$$\int \frac{1}{(x^3-3x^2+7x-5)^{1/3}}\,dx$$

Optimal(type 3, 67 leaves, ? steps):

$$\frac{\ln(1-x)}{4} - \frac{3\ln\left(1-x+(x^3-3x^2+7x-5)^{1/3}\right)}{4} + \frac{\arctan\left(\frac{\sqrt{3}}{3} + \frac{2\,(x-1)\sqrt{3}}{3\,(x^3-3x^2+7x-5)^{1/3}}\right)\sqrt{3}}{2}$$

Result(type 8, 17 leaves):

$$\int \frac{1}{(x^3-3x^2+7x-5)^{1/3}}\,dx$$

Problem 15: Unable to integrate problem.

$$\int \frac{2-(1+k)\,x}{\left((1-x)\,x\,(-kx+1)\right)^{1/3}\,(1-(1+k)\,x)}\,dx$$

Optimal(type 3, 86 leaves, ? steps):

$$\frac{\ln(x)}{2\,k^{1/3}}+\frac{\ln(1-(1+k)\,x)}{2\,k^{1/3}}-\frac{3\ln\left(-k^{1/3}x+\left((1-x)\,x\,(-kx+1)\right)^{1/3}\right)}{2\,k^{1/3}}+\frac{\arctan\left(\frac{\left(1+\frac{2\,k^{1/3}x}{\left((1-x)\,x\,(-kx+1)\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{k^{1/3}}$$

Result(type 8, 36 leaves):

$$\int \frac{2-(1+k)\,x}{\left((1-x)\,x\,(-kx+1)\right)^{1/3}\,(1-(1+k)\,x)}\,dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{c\,x^2+b\,x+a}{(x^2-x+1)\,(-x^3+1)^{1/3}}\,dx$$

Optimal(type 3, 392 leaves, 19 steps):

$$\frac{(a+b)\ln\left((1-x)\,(1+x)^2\right)2^{2/3}}{24}-\frac{(a-c)\ln\left(x^3+1\right)2^{2/3}}{12}-\frac{(b+c)\ln\left(x^3+1\right)2^{2/3}}{12}+\frac{(a+b)\ln\left(1+\frac{2^{2/3}(1-x)^2}{(-x^3+1)^{2/3}}-\frac{2^{1/3}(1-x)}{(-x^3+1)^{1/3}}\right)2^{2/3}}{12}$$
$$-\frac{(a+b)\ln\left(1+\frac{2^{1/3}(1-x)}{(-x^3+1)^{1/3}}\right)2^{2/3}}{6}+\frac{(b+c)\ln\left(2^{1/3}-(-x^3+1)^{1/3}\right)2^{2/3}}{4}+\frac{(a-c)\ln\left(-2^{1/3}x-(-x^3+1)^{1/3}\right)2^{2/3}}{4}$$
$$+\frac{c\ln\left(x+(-x^3+1)^{1/3}\right)}{2}-\frac{(a+b)\ln\left(-1+x+2^{2/3}(-x^3+1)^{1/3}\right)2^{2/3}}{8}+\frac{(a+b)\arctan\left(\frac{\left(1-\frac{2\,2^{1/3}(1-x)}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)2^{2/3}\sqrt{3}}{6}$$
$$+\frac{(a+b)\arctan\left(\frac{\left(1+\frac{2^{1/3}(1-x)}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)2^{2/3}\sqrt{3}}{12}-\frac{c\arctan\left(\frac{\left(1-\frac{2\,x}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$
$$-\frac{(a-c)\arctan\left(\frac{\left(1-\frac{2\,2^{1/3}x}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)2^{2/3}\sqrt{3}}{6}+\frac{(b+c)\arctan\left(\frac{\left(1+2^{2/3}(-x^3+1)^{1/3}\right)\sqrt{3}}{3}\right)2^{2/3}\sqrt{3}}{6}$$

Result(type 8, 32 leaves):

$$\int \frac{c\,x^2+b\,x+a}{(x^2-x+1)\,(-x^3+1)^{1/3}}\,dx$$

Problem 18: Unable to integrate problem.

$$\int \frac{1}{x\,(3\,x^2-6\,x+4)^{1/3}}\,dx$$

Optimal(type 3, 76 leaves, 1 step):

$$-\frac{\ln(x)\,2^{1/3}}{4}+\frac{\ln\left(6-3\,x-3\,2^{1/3}\,(3\,x^2-6\,x+4)^{1/3}\right)\,2^{1/3}}{4}+\frac{\arctan\left(-\frac{\sqrt{3}}{3}-\frac{2^{2/3}\,(2-x)\,\sqrt{3}}{3\,(3\,x^2-6\,x+4)^{1/3}}\right)\,2^{1/3}\,\sqrt{3}}{6}$$

Result(type 8, 18 leaves):

$$\int \frac{1}{x\,(3\,x^2-6\,x+4)^{1/3}}\,dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{(-x^3+1)^{1/3}}{1+x}\,dx$$

Optimal(type 3, 381 leaves, 25 steps):

$$(-x^3+1)^{1/3}-\frac{2^{1/3}\ln(x^3+1)}{3}+\frac{\ln\left(2^{2/3}+\frac{x-1}{(-x^3+1)^{1/3}}\right)\,2^{1/3}}{6}-\frac{\ln\left(1+\frac{2^{2/3}\,(1-x)^2}{(-x^3+1)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)\,2^{1/3}}{6}$$
$$+\frac{2^{1/3}\ln\left(1+\frac{2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)}{3}-\frac{\ln\left(2\,2^{1/3}+\frac{(1-x)^2}{(-x^3+1)^{2/3}}+\frac{2^{2/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)\,2^{1/3}}{12}+\frac{\ln\left(2^{1/3}-(-x^3+1)^{1/3}\right)\,2^{1/3}}{2}$$
$$-\frac{\ln\left(-x-(-x^3+1)^{1/3}\right)}{2}+\frac{\ln\left(-2^{1/3}\,x-(-x^3+1)^{1/3}\right)\,2^{1/3}}{2}+\frac{2^{1/3}\arctan\left(\frac{\left(1-\frac{2\,2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$
$$+\frac{\arctan\left(\frac{\left(1+\frac{2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\,2^{1/3}\,\sqrt{3}}{6}-\frac{\arctan\left(\frac{\left(1-\frac{2\,x}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{2^{1/3}\arctan\left(\frac{\left(1-\frac{2\,2^{1/3}\,x}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$
$$-\frac{2^{1/3}\arctan\left(\frac{\left(1+2^{2/3}\,(-x^3+1)^{1/3}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$

Result(type 8, 58 leaves):

$$-\frac{x^3-1}{(-x^3+1)^{2/3}}+\frac{\left(\int \frac{x^2+1}{(1+x)\left((x^3-1)^2\right)^{1/3}}\,dx\right)\left((x^3-1)^2\right)^{1/3}}{(-x^3+1)^{2/3}}$$

```
Problem 22: Result more than twice size of optimal antiderivative.
```

$$\int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

```
Optimal(type 3, 41 leaves, 1 step):
```

$$\frac{\arctan\left(\frac{x(x^2+1)}{\sqrt{-x^4+1}}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{x(-x^2+1)}{\sqrt{-x^4+1}}\right)}{2}$$

```
Result(type 3, 99 leaves):
```

$$-\frac{\arctan\left(\frac{\sqrt{-x^4+1}}{x}+1\right)}{4}+\frac{\arctan\left(-\frac{\sqrt{-x^4+1}}{x}+1\right)}{4}-\frac{\ln\left(\frac{\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}+1}{\frac{\sqrt{-x^4+1}}{x}+\frac{-x^4+1}{2x^2}+1}\right)}{8}$$

```
Problem 23: Unable to integrate problem.
```

$$\int \frac{bx+a}{(-x^2+2)(x^2-1)^{1/4}}\,dx$$

```
Optimal(type 3, 62 leaves, 7 steps):
```

$$-b\arctan\left((x^2-1)^{1/4}\right)+b\operatorname{arctanh}\left((x^2-1)^{1/4}\right)+\frac{a\arctan\left(\frac{x\sqrt{2}}{2(x^2-1)^{1/4}}\right)\sqrt{2}}{4}+\frac{a\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2(x^2-1)^{1/4}}\right)\sqrt{2}}{4}$$

```
Result(type 8, 24 leaves):
```

$$\int \frac{bx+a}{(-x^2+2)(x^2-1)^{1/4}}\,dx$$

```
Problem 24: Unable to integrate problem.
```

$$\int \frac{1}{(-x^2+1)^{1/3}(x^2+3)}\,dx$$

```
Optimal(type 3, 81 leaves, 1 step):
```

$$-\frac{\operatorname{arctanh}(x)\,2^{1/3}}{12}+\frac{\operatorname{arctanh}\left(\frac{x}{1+2^{1/3}\,(-x^2+1)^{1/3}}\right)2^{1/3}}{4}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)2^{1/3}\sqrt{3}}{12}+\frac{\arctan\left(\frac{\left(1-2^{1/3}\,(-x^2+1)^{1/3}\right)\sqrt{3}}{x}\right)2^{1/3}\sqrt{3}}{12}$$

Result(type 8, 19 leaves):

$$\int \frac{1}{(-x^2+1)^{1/3}\,(x^2+3)}\,dx$$

Problem 25: Unable to integrate problem.

$$\int \frac{1}{(-x^2+3)\,(x^2+1)^{1/3}}\,dx$$

Optimal(type 3, 77 leaves, 1 step):

$$-\frac{\arctan(x)\,2^{1/3}}{12}+\frac{\arctan\left(\frac{x}{1+2^{1/3}\,(x^2+1)^{1/3}}\right)2^{1/3}}{4}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)2^{1/3}\sqrt{3}}{12}-\frac{\operatorname{arctanh}\left(\frac{\left(1-2^{1/3}\,(x^2+1)^{1/3}\right)\sqrt{3}}{x}\right)2^{1/3}\sqrt{3}}{12}$$

Result(type 8, 19 leaves):

$$\int \frac{1}{(-x^2+3)\,(x^2+1)^{1/3}}\,dx$$

Problem 26: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+x-\sqrt{3}}{\left(1+x+\sqrt{3}\right)\sqrt{-4+x^4+4x^2\sqrt{3}}}\,dx$$

Optimal(type 3, 47 leaves, 2 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\left(1+x-\sqrt{3}\right)^2}{\sqrt{-9+6\sqrt{3}}\,\sqrt{-4+x^4+4x^2\sqrt{3}}}\right)\sqrt{-3+2\sqrt{3}}}{3}$$

Result(type 4, 326 leaves):

$$\frac{\sqrt{1-\left(\frac{\sqrt{3}}{2}-1\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\ \operatorname{EllipticF}\left(x\left(\frac{I\sqrt{3}}{2}-\frac{I}{2}\right), I\sqrt{1+4\sqrt{3}\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{I\sqrt{3}}{2}-\frac{I}{2}\right)\sqrt{-4+x^4+4x^2\sqrt{3}}}-2\sqrt{3}\left(\vphantom{\frac{1}{2}}\right.$$

$$-\frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}\,(-1-\sqrt{3})^2-8+4x^2\sqrt{3}+2x^2(-1-\sqrt{3})^2}{2\sqrt{(-1-\sqrt{3})^4+4\sqrt{3}\,(-1-\sqrt{3})^2-4}\,\sqrt{-4+x^4+4x^2\sqrt{3}}}\right)}{2\sqrt{(-1-\sqrt{3})^4+4\sqrt{3}\,(-1-\sqrt{3})^2-4}}$$

$$-\frac{\sqrt{1-\left(\frac{\sqrt{3}}{2}-1\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\sqrt{3}}{2}-1}\,x,\frac{1}{\left(\frac{\sqrt{3}}{2}-1\right)(-1-\sqrt{3})^2},\frac{\sqrt{1+\frac{\sqrt{3}}{2}}}{\sqrt{\frac{\sqrt{3}}{2}-1}}\right)}{\sqrt{\frac{\sqrt{3}}{2}-1}\,(-1-\sqrt{3})\,\sqrt{-4+x^4+4x^2\sqrt{3}}}\Bigg)$$

Problem 27: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1+x+\sqrt{3}}{(1+x-\sqrt{3})\sqrt{-4+x^4-4x^2\sqrt{3}}}\,dx$$

Optimal(type 3, 45 leaves, 2 steps):

$$-\frac{\arctan\left(\frac{(1+x+\sqrt{3})^2}{\sqrt{9+6\sqrt{3}}\,\sqrt{-4+x^4-4x^2\sqrt{3}}}\right)\sqrt{3+2\sqrt{3}}}{3}$$

Result(type 4, 310 leaves):

$$\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{3}}{2}+1\right)x^2}\,\operatorname{EllipticF}\left(x\left(\frac{I}{2}+\frac{I\sqrt{3}}{2}\right),I\sqrt{1-4\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)}\right)}{\left(\frac{I}{2}+\frac{I\sqrt{3}}{2}\right)\sqrt{-4+x^4-4x^2\sqrt{3}}}+2\sqrt{3}\Bigg($$

$$-\frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}\left(\sqrt{3}-1\right)^2-8-4x^2\sqrt{3}+2x^2\left(\sqrt{3}-1\right)^2}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}\sqrt{-4+x^4-4x^2\sqrt{3}}}\right)}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}}$$

$$-\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{3}}{2}+1\right)x^2}\operatorname{EllipticPi}\left(\sqrt{-1-\frac{\sqrt{3}}{2}}\,x,\frac{1}{\left(-1-\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}-1\right)^2},\frac{\sqrt{-\frac{\sqrt{3}}{2}+1}}{\sqrt{-1-\frac{\sqrt{3}}{2}}}\right)}{\sqrt{-1-\frac{\sqrt{3}}{2}}\left(\sqrt{3}-1\right)\sqrt{-4+x^4-4x^2\sqrt{3}}}\Bigg)$$

Problem 28: Unable to integrate problem.

$$\int\frac{x^2}{\left(-x^3+1\right)^{1/3}\left(x^3+1\right)}\,dx$$

Optimal(type 3, 62 leaves, 5 steps):

$$-\frac{\ln\left(x^3+1\right)2^{2/3}}{12}+\frac{\ln\left(2^{1/3}-\left(-x^3+1\right)^{1/3}\right)2^{2/3}}{4}+\frac{\arctan\left(\frac{\left(1+2^{2/3}\left(-x^3+1\right)^{1/3}\right)\sqrt{3}}{3}\right)2^{2/3}\sqrt{3}}{6}$$

Result(type 8, 22 leaves):

$$\int\frac{x^2}{\left(-x^3+1\right)^{1/3}\left(x^3+1\right)}\,dx$$

Problem 29: Unable to integrate problem.

$$\int\frac{1+x}{\left(x^2-x+1\right)\left(-x^3+1\right)^{1/3}}\,dx$$

Optimal(type 3, 109 leaves, ? steps):

$$\frac{\ln\left(1+\frac{2^{2/3}\left(1-x\right)^2}{\left(-x^3+1\right)^{2/3}}-\frac{2^{1/3}\left(1-x\right)}{\left(-x^3+1\right)^{1/3}}\right)2^{2/3}}{4}-\frac{\ln\left(1+\frac{2^{1/3}\left(1-x\right)}{\left(-x^3+1\right)^{1/3}}\right)2^{2/3}}{2}+\frac{\arctan\left(\frac{\left(1-\frac{2\,2^{1/3}\left(1-x\right)}{\left(-x^3+1\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}\,2^{2/3}}{2}$$

Result(type 8, 25 leaves):

$$\int \frac{1+x}{(x^2-x+1)\,(-x^3+1)^{1/3}}\,dx$$

Problem 30: Unable to integrate problem.

$$\int \frac{(1+x)^2}{(-x^3+1)^{1/3}\,(x^3+1)}\,dx$$

Optimal(type 3, 109 leaves, ? steps):

$$\frac{\ln\left(1+\frac{2^{2/3}\,(1-x)^2}{(-x^3+1)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)2^{2/3}}{4}-\frac{\ln\left(1+\frac{2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)2^{2/3}}{2}+\frac{\arctan\left(\frac{\left(1-\frac{2\,2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}\,2^{2/3}}{2}$$

Result(type 8, 24 leaves):

$$\int \frac{(1+x)^2}{(-x^3+1)^{1/3}\,(x^3+1)}\,dx$$

Problem 31: Unable to integrate problem.

$$\int \frac{(x^2-x+1)\,(-x^3+1)^{2/3}}{x^3+1}\,dx$$

Optimal(type 5, 138 leaves, 6 steps):

$$\frac{(-x^3+1)^{2/3}}{2}+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}-\frac{\ln\left((1-x)\,(1+x)^2\right)2^{2/3}}{4}-\frac{\ln\left(x+(-x^3+1)^{1/3}\right)}{2}+\frac{3\ln\left(-1+x+2^{2/3}\,(-x^3+1)^{1/3}\right)2^{2/3}}{4}$$
$$+\frac{\arctan\left(\frac{\left(1-\frac{2x}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\arctan\left(\frac{\left(1+\frac{2^{1/3}\,(1-x)}{(-x^3+1)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}\,2^{2/3}}{2}$$

Result(type 8, 39 leaves):

$$-\frac{x^3-1}{2\,(-x^3+1)^{1/3}}+\int \frac{x^2+1}{(1+x)\,(-x^3+1)^{1/3}}\,dx$$

Problem 32: Unable to integrate problem.

$$\int \frac{(-x^3+1)^{1/3}}{x^3+1}\,dx$$

Optimal(type 3, 213 leaves, 14 steps):

$$\frac{\ln\left(2^{2/3}+\frac{x-1}{\left(-x^{3}+1\right)^{1/3}}\right)2^{1/3}}{6}-\frac{\ln\left(1+\frac{2^{2/3}\left(1-x\right)^{2}}{\left(-x^{3}+1\right)^{2/3}}-\frac{2^{1/3}\left(1-x\right)}{\left(-x^{3}+1\right)^{1/3}}\right)2^{1/3}}{6}+\frac{2^{1/3}\ln\left(1+\frac{2^{1/3}\left(1-x\right)}{\left(-x^{3}+1\right)^{1/3}}\right)}{3}$$

$$-\frac{\ln\left(2\,2^{1/3}+\frac{\left(1-x\right)^{2}}{\left(-x^{3}+1\right)^{2/3}}+\frac{2^{2/3}\left(1-x\right)}{\left(-x^{3}+1\right)^{1/3}}\right)2^{1/3}}{12}+\frac{2^{1/3}\arctan\left(\frac{\left(1-\frac{2\,2^{1/3}\left(1-x\right)}{\left(-x^{3}+1\right)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$

$$+\frac{\arctan\left(\frac{\left(1+\frac{2^{1/3}\left(1-x\right)}{\left(-x^{3}+1\right)^{1/3}}\right)\sqrt{3}}{3}\right)2^{1/3}\sqrt{3}}{6}$$

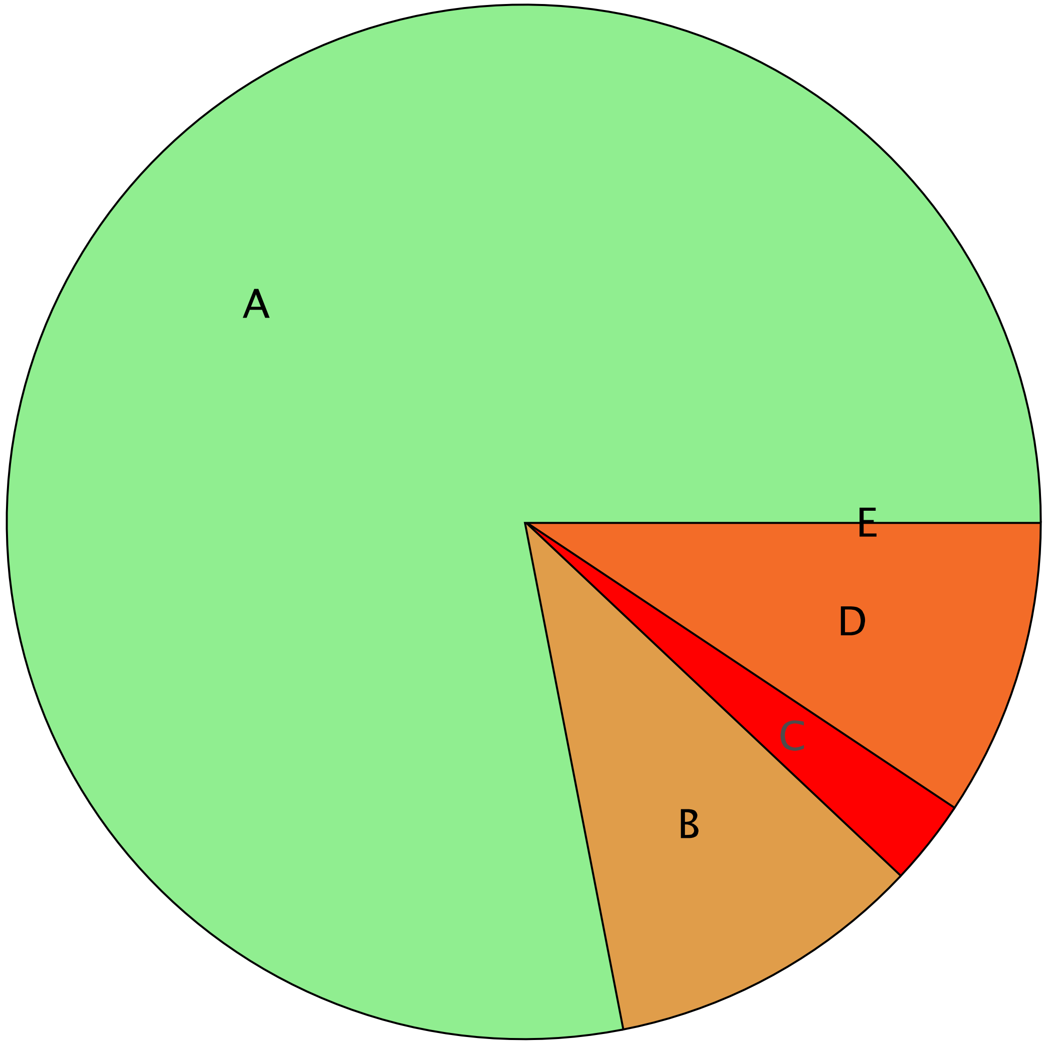
Result(type 8, 19 leaves):

$$\int\frac{\left(-x^{3}+1\right)^{1/3}}{x^{3}+1}\,dx$$

Test results for the 3 problems in "Wester Problems.txt"

Summary of Integration Test Results

524 integration problems



A - 409 optimal antiderivatives
B - 52 more than twice size of optimal antiderivatives
C - 14 unnecessarily complex antiderivatives
D - 49 unable to integrate problems
E - 0 integration timeouts